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Optimal Fuel Strategy for Portfolio Profit Maximization

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Martin Kragelund

*Optimal Fuel Strategy for
Portfolio Profit Maximization*

Optimal Fuel Strategy for Portfolio Profit Maximization
Ph.D. thesis

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| Preface and Acknowledgements

This thesis is submitted as a collection of papers in partial fulfillment of the requirements for a Doctor of Philosophy at the Section of Automation and Control, Department of Electronic Systems, Aalborg University, Denmark. The work has been carried out in the period November 2006 to November 2009 under the supervision of Professor Rafał Wisniewski.

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I would like to thank everyone at the Section of Automation and Control, who has, directly or indirectly, helped during the three years of my PhD-study, and in particular John Leth and my supervisor Rafał Wisniewski, who have helped me in times of trouble with discussions and guidance. Furthermore, I would like to thank Dong Energy for providing measurement data, insight into the power market, and an initial problem, which all have helped make this research possible.

I was, during the spring 2009, visiting The Royal Institute of Technology in Stockholm, Sweden and I would like to thank everyone at the Division of Optimization and Systems Theory, and especially Ulf Jönsson for making the visit possible, taking interest and participating in my research.

Aalborg University, November 2009
Martin Kragelund

| Abstract

Traditionally, process control systems are designed, verified, and implemented without consideration of possible extensions of the system in the future. Redesigning a control system often involve large cost and therefore, many currently running control systems do not behave even close to optimal. Furthermore, the sensors and actuators of a control system are often assumed given when the system design is begun. However, the selection of sensors and actuators dictates the performance of the system, which for many companies is related to the profit of the company. This problem is addressed by the Plug and Play Process control system.

This thesis establishes a model of the profit of a power plant capable of using three different fuel systems, which are a coal system, a gas system, and a oil system and a strategy for maximizing the profit of the power plant by utilizing the three fuel systems is developed. The profit can be described as the integral over time of a function of the system states and some market prices. These prices have been identified and models of them have been established using data from the Nordic power marketplace, Nord Pool.

The maximization of the profit has been performed using three different approaches ranging from a search of the input space over static optimization to Pontryagin's maximum principle. The first approach did not consider the dynamics of the different fuel systems, instead focus was placed on developing models of the value of the business objectives of the power plant and relating them to the different fuel systems.

The second approach utilized a notion from production economics where each of the products (business objectives in this work) is given a price, i.e., the price data was separated from the models developed in the first approach and thus allowing for price changes over time. For a given time static optimization was used for maximization.

The final approach included the dynamics of the different fuel systems in the optimization and both an open loop control and a feedback control strategy were developed using discrete optimization and maximum principle, respectively. In the open loop control the profit function was converted to discrete time and the optimization was performed using quadratic programming, where the production reference tracking was included as a constraint. Removing the tracking from the constraint and including it as a tracking error in the objective function made the problem less complex and a continuous solution was obtained using Pontryagin's maximum principle.

| Synopsis

Proceskontrollsystemer er traditionelt designet, verificeret og implementeret uden overvejelser omkring mulige udvidelser i fremtiden. Hvis et kontrolsystem skal redesignes involverer det ofte store omkostninger og derfor er der mange kørende kontrolsystemer som ikke opererer optimalt. *Plug and Play Process Control* projektet vil afhjælpe dette problem. Derudover er sensorer og aktuatorer som oftest antaget at være kendt når et systemdesign er påbegyndt. Dette er dog et problem da valget af sensorer og aktuatorer sætte en begrænsning for hvilken ydelse der kan forventes af systemet og dermed også firmaets profit.

Denne afhandling opstiller en indtjeningsmodel for et kraftværk som er i stand til at benytte tre forskellige brændselssystemer. De forskellige brændselssystemer er kul-, gas- og olie-systemer. Derudover udvikles der også en strategi for at maksimere kraftværkets profit ved at benytte de forskellige brændsler optimalt. Kraftværkets profit kan beskrives som et integrale over tid af en funktion af systemets tilstande og relevante markedspriser. Disse priser er blevet identificeret og modeller for dem er blevet dannet med udgangspunkt i det nordiske elektricitetsmarked kaldet Nord Pool.

Tre forskellige fremgangsmåder er blevet benyttet til profitmaksimeringen og de varierer fra søgning efter den optimale input og statisk optimering til Pontryagins maksimumsprincip. Den første fremgangsmåde tog ikke højde for dynamikken af de forskellige brændselssystemer, men i stedet var fokus på at udvikle modeller af de forskellige forretningsmål for kraftværket og relaterer dem til de forskellige brændselssystemer.

Den anden fremgangsmåde brugte et begreb fra produktionsøkonomi hvor hvert produkt (forretningsmål i dette arbejde) er givet en pris. Det vil sige at priser blev adskilt fra modellerne udviklet i den første fremgangsmåde og dette gør det muligt at ændre priserne over tid. Til et givet tidspunkt blev statisk optimering brugt til at maksimere profitten.

Den sidste fremgangsmåde inkluderede dynamikken af de forskellige brændselssystemer i optimeringen og både en åbensløjfe og feedback løsning var udviklet ved brug af diskret optimering og maksimumsprincip respektivt. I åbensløjfekonrolstrategien var profitfunktionen konverteret til diskret tid og optimeringen var udført ved hjælp af lineær programmering hvor produktionsreferencen blev inkluderet som en sidebetingelse. Ved at fjerne sidebetingelsen og i stedet inkluderer referencen i kostfunktionen som en referencefejl blev optimeringsproblemet mindre kompleks og en kontinuer løsning blev fundet ved hjælp af Pontryagins maksimumsprincip.

1 | Introduction

In a time where much focus is placed on money and profit, this thesis will present an idea of formulation the objective when scheduling production and usage of actuators as profit maximization.

In this work we focus on a power plant which is capable of using three different fuel systems and identify the measures which influence the economics of the company.

This chapter describes the motivation of this work, and in particular the Plug and Play Process Control project, which this is part of. Then an overview of the state of the art within sensor/actuator selection and economics of control as well as optimal control is given and finally in the end of this chapter an outline of this thesis will be presented.

1.1 Motivation

The vision of the Plug and Play Process Control (P3C) project is to develop a new control paradigm, which is capable of adapting to changes introduced in industrial processes. The vision is a control system which detects when devices are added or removed from the system and automatically reconfigures the control such that optimum performance is obtained. An example of P3C could be a heating system in a regular home consisting e.g. of a number of standard heaters and a newly added heat exchanger. The home owner might after the installation of the heat exchanger find that temperature in some areas in his home are not as comfortable as he wished. Therefore, he installs a number of new temperature sensors and the P3C system should then automatically detect these new sensors and incorporate their measurements in the control system to improve the indoor climate (for more information of the vision of P3C see [P3C homepage, 2009], [Stoustrup, 2009], and [Bendtsen et al., 2008]). Five companies participate in the P3C-project; Skov, DONG Energy, Danfoss, Grundfos, and FLSmidth Automation. Each of the companies provides a case study of their Plug and Play problem.

The work of the P3C project has been divided into 6 work packages dealing with different aspects of the project. These 6 work packages are introduced in the following.

Work Package 1 - Integration of hardware, networks and protocols for flexible control systems. The objective of this work package is to develop the infrastructure needed for the project, i.e., investigate what kind of communication scheme is need and how to identify when new hardware is introduced.

Work Package 2 - Correlation based sensor/actuator awareness - This work package deals with identifying what and where the introduced hardware is, i.e., identify if it

is a new sensor or an actuator and what kind of changes can be obtained/measured when the new hardware is in use. Furthermore, the model of the system should be expanded to include the new hardware (this could be either a black box or white box model depending on the situation). For further reference see [Knudsen et al., 2008], [Knudsen, 2009a], [Knudsen, 2009b], and [Bendtsen and Trangbaek, 2008].

Work Package 3 - Structurally based reconfiguration - The objective of this work packages is to develop algorithms which incorporate the new hardware in old controller, e.g. use an additional sensor to obtain a better estimate of the current measurement. For further reference see [Trangbaek et al., 2008], [Trangbaek et al., 2009], [Stoustrup et al., 2009], and [Trangbaek, 2009].

Work Package 4 - Model-based control performance optimization through flexible sensor/actuator configuration - This work package is similar to work package 3, however, the restructuring of the controller will in this work package be based on explicit models of the complete system and performance optimization. The performance of new hardware needs to be specified such that the optimization knows how it behaves in accordance with the overall objective. [Michelsen et al., 2008], [Michelsen et al., 2009], and [Michelsen and Trangbaek, 2009]

Work Package 5 - Survivability and performability measures - This work package is not part of an complete online P3C-system, as the objective is to evaluate which new hardware should be plugged into the system to obtain better performance (if possible). The work of this work package is currently in the hands of the reader and will be outlined in more detail later.

Work Package 6 - Decentralized event-based networked nonlinear control for plug-and-play process control - The objective of this work package is to investigate the event-based and decentralized perspective of P3C. The option of adding and removing hardware in an existing system will often call for using wireless components, which have limited communication possibilities in some way either due to power or noise effects. For further reference see [Persis and Kallesøe, 2008], and [Persis and Kallesøe, 2009]

This work concerns with work package 5 and profit maximization of a power plant. The basic idea of this work is to automatically propagate the business objectives of a company to the selection of sensors and actuators - usually, the goal of a company is to maximize profit. In this work it is suggested to maximize the profit by selecting the optimal sensors and actuators. However, the automatic propagation of requirements is difficult and usually need knowledge of the system [Leveson et al., 1994] and [Foss, 1973]. The later also conclude that many performance measures exist for process control system and which to use is a designer's choice but no obvious method exists.

"Present practice is based largely on direct observations and experience gained by operators and plant management." [Mesarovic, 1970]

Compared to the other work package in the P3C-project, work package 5 is used offline to evaluate which sensors and/or actuators should be used, i.e., it answers the question what (and when) to plug? That is given a profit function of the company and possible new actuators this work packages will tell if and when the new actuators should be used. This will be explained in more detail in Chapter 2

1.2 State of the Art and Background

The line of work in this thesis bears resemblance to sensor/actuator placement and plant-wide control but also the economic perspective of implementing and operating a plant with a certain sensor and actuator configuration is of interest to this work. Important for this work is also the power market, which dictates how a power plant earns money. Furthermore, this work leans on methods from optimization and in particular optimal control. In the following the state of the art of these subject will be described.

1.2.1 Structural Sensor and Actuator Selection

Methods which are used to structure the controller by selecting sensors and actuators are considered in this section. The term structural sensor and actuator selection refers to the fact the these methods should only rely on the plant and not the implemented controller.

In [Skogestad and Postlethwaite, 2005, chapter 1] the process of control system design is presented as 14 steps, which are

1. Study the system (process, plant) to be controlled and obtain initial information about the control objectives.
2. Model the system and simplify the model, if necessary.
3. Scale the variables and analyze the resulting model; determine its properties.
4. Decide which variables are to be controlled (controlled outputs).
5. Decide on the measurements and manipulated variables: what sensors and actuators will be used and where will they be placed?
6. Select the control configuration.
7. Decide on the type of controller to be used.
8. Decide on performance specifications, based on the overall control objectives.
9. Design a controller.
10. Analyze the resulting controlled system to see if the specifications are satisfied; and if they are not satisfied modify the specifications or the type of controller.
11. Simulate the resulting controlled system, on either a computer or a pilot plant.
12. Repeat from step 2, if necessary.
13. Choose hardware and software and implement the controller.
14. Test and validate the control system, and tune the controller on-line, if necessary.

It is concluded in [Skogestad and Postlethwaite, 2005, chapter 10] that usually only step 9, which is controller design, is considered by the academia. Furthermore, steps 4, 5, and 6 are generally overlooked when control systems are developed, i.e., which variables should be controlled, which sensors and actuators should be used, and which controller structure should be utilized is often assumed known in control theory. However, in practice the selection of those quantities are often of importance as bad choices often limit the performance [van de Wal and de Jager, 2001] and it is a problem faced by the control engineer of a given plant [Foss, 1973].

In [Foss, 1973] a discussion is given about the gap between control theory and applications, especially control of chemical processes and it is stated that the theory has to

meet up with practice. It is concluded that methods do exist for designing SISO control and that SISO control loops can be found by known method. One such method, the relative gain array (RGA), is presented in [Bristol, 1966]. The RGA can be used to pair inputs and outputs in a MIMO system to enable decentralized control, i.e., in a way it can be used to select control structure. The RGA is defined as

$$RGA(G) = G \times (G^{-1})^T,$$

for a plant with a square transfer matrix¹ G . The pairing of input and output should be performed such that the value of the diagonal entries of RGA is close to one in the frequency band of interest and negative values should be avoided. The relative gain array has especially been used when designing control system for chemical processes e.g. in [Papadourakis et al., 1987] RGA is used to assess how to pair inputs and outputs in subsystem in a chemical plant. A similar method involves the singular value decomposition (SVD) of the transfer matrix which can be used to determine how different input directions influence the output.

Recent contributions [van de Wal and de Jager, 2001], [Skogestad and Larsson, 1998], and [Stephanopoulos and Ng, 2000] have shown that progress in sensor and actuator selection and plant wide control in recent years took place. Especially within specific areas methods have been developed, e.g. flexible structures within aerospace (see [Padula and Kincaid, 1999]). In [van de Wal and de Jager, 2001] eight different selection criteria are described and they are assessed according to some desirable properties which are

Well-founded: The theory should be sound and complete.

Efficient: The expected computational effort should be small.

Effective: The selection method should be both necessary and sufficient.

Generally applicable: Easy to use the method on other problems.

Rigorous: A small number of candidates should be selected.

Quantitative: It should be possible to measure how “far” two sets of actuators and sensors are from each other.

Controller independent: The controller structure should be selected after the sensors and actuators are chosen.

Direct: The selection method should be direct, meaning that the different sensor-actuator sets should not be evaluated one by one.

The reviewed selection criteria in [van de Wal and de Jager, 2001] can be divided into three categories; controllability and observability measures (both qualitative and quantitative and with and without noise), efficiency measures (minimization of input/output energy), and robust methods (robust stability and robust performance). Common for all the presented selection methods is that the problem grows exponentially with the number of possible inputs and outputs. Therefore, [van de Wal and de Jager, 2001] suggests two methods which can limit the search if the method is not direct: (A) eliminate input-outputs which do not improve control and (B) add inputs/outputs which do not worsen

¹The Relative Gain Array can be extended to non-square plants see [Skogestad and Postlethwaite, 2005, Appendix A] for further comments

control. Most available methods are concluded to be indirect, i.e., each combination of sensors and actuators needs to be checked individually.

1.2.2 Economics of Control

In addition to the previous section, economic considerations are also important when a system is instrumented. This subject had some attention in the late 70s and it has gained focus again, lately.

Industrial process control system is often built according to the scheme illustrated in Figure 1.1. Typically, the bottom level consists of single input - single output control loops of the individual actuators where detailed and complex dynamic are present. The closed loops at the bottom level makes it possible at the middle level to modeled the bottom layer with linear dynamics and develop multiple input - multiple output controllers to coordinate the different actuators in each process. At the top level the processes is usually considered without dynamics and economics are considered here as well as constraints present in the system. Thus, economic optimization of the complete plant is performed at the top level to ensure optimal operation by generating optimal setpoints wrt the plant economics.

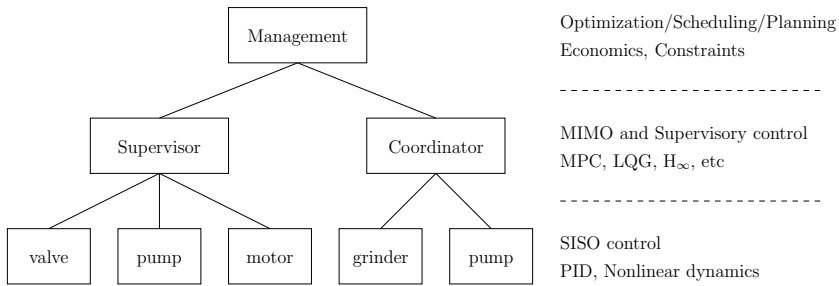


Figure 1.1: Illustration of how industrial process control systems are built in a hierarchical structure.

The line of thinking in Figure 1.1 is described in [Mesarovic, 1970], where hierarchical structures are identified. Basically three different hierarchies are introduced, namely stratum, complexity-layers and organizational hierarchy-echelons. Furthermore, some examples of applications, where hierarchies are used, are presented in [Mesarovic, 1970], e.g. power plant grid utilization, ethylene, and steel production. All of these applications and the different hierarchy structures have economic optimization as a common feature at the top level. It is concluded that total system performance can be improved by using computer control but it is not more accurate control which is called for. Instead improvements in plant operations facilitates “*substantial economic advantages*” [Mesarovic, 1970].

Optimization of hierarchical systems as presented in [Mesarovic, 1970] are considered by [Findeisen et al., 1980] and [Bryds et al., 1989] where the control system can be separated into a number of subsystem. Optimal setpoints for each subsystem are considered in the optimization and often from an economic perspective. A hierarchical structure with economic considerations on the top level is also presented

in [Skogestad and Postlethwaite, 2005, Chapter 10]. Thus, it is concluded that economic considerations are important in control system design.

More recently hierarchical systems have been used by [Rantzer, 2009] for decentralized control, where the objective function is divided into a local problem for each subsystem and a simple global problem, which guarantee the original objective is fulfilled. Furthermore, local algorithms are given for determining how far the local solution is from the optimal.

The economic considerations when designing control systems have also gained more attention lately e.g. [Lu and Skelton, 1999a] and [Lu and Skelton, 1999b] introduced the Economic Design Problem: *"for a given performance requirement, design the feedback control law and distribute signal-to-noise ratios among the instruments (sensors, actuators, A/D, D/A conversion, control processing) such that the instrumentation cost is minimized without compromising the system performance."* Thus, it is assumed that the cost of a sensor/actuator is proportional to the signal-to-noise ratio. In [Skelton and Li, 2004] this idea has been applied to control of a flexible structure with 18 degrees of freedom and 21 possible sensors and actuators. The accuracy of the output is specified and the proposed iterative algorithm chooses the least precise sensors and actuators capable of delivering the specified output accuracy. The method in [Skelton and Li, 2004] has in [Li et al., 2006] been converted to a set of LMIs and thus a convex optimization problem.

The cost to sensors and actuators are often minor compared to the operational cost of a plant, which in many cases is the main economic concern [Skogestad and Larsson, 1998]. It is furthermore stated that "The economics of plant operation are usually determined by steady-state issues," which indicates that an optimum condition for dynamic economic control of a plant is missing.

1.2.3 Electrical Power Market and Power Plants

In this section different aspects of the electrical power market and related work will be presented. The chosen subjects are conventional power plants, production economics, power plant operations, the power market. This section will end with some related work performed on hydro power plants in Norway.

Traditional thermal power plants, i.e., coal, gas, or oil fired power plants, have been studied in detail e.g. in [Flynn, 2003]. The thermal power plant is illustrated in Figure 1.2 and basically function by burning a fuel in the boiler which evaporates water to steam under high pressure. The steam then drives a turbine generating electrical power which is delivered to the electrical grid. A thermal power plant is modeled by first principle in [Andersen et al., 2005], where the considered fuel is coal dust which arise from four coal mills grinding the raw coal.

The detailed model in [Andersen et al., 2005] was used to establish an observer for the flow of coal into the boiler to improve the control of the coal mills. Simpler models for system control are presented in [Edlund et al., 2009b], where the different possible methods for changing the output from the complete portfolio of DONG Energy in Denmark are described. Each of these possible methods of changing the output is in [Edlund et al., 2009b] denoted an effectuator and a model is derived. An example of an effectuator is the boiler load in a thermal power plant which can be modeled as a 3rd order system.

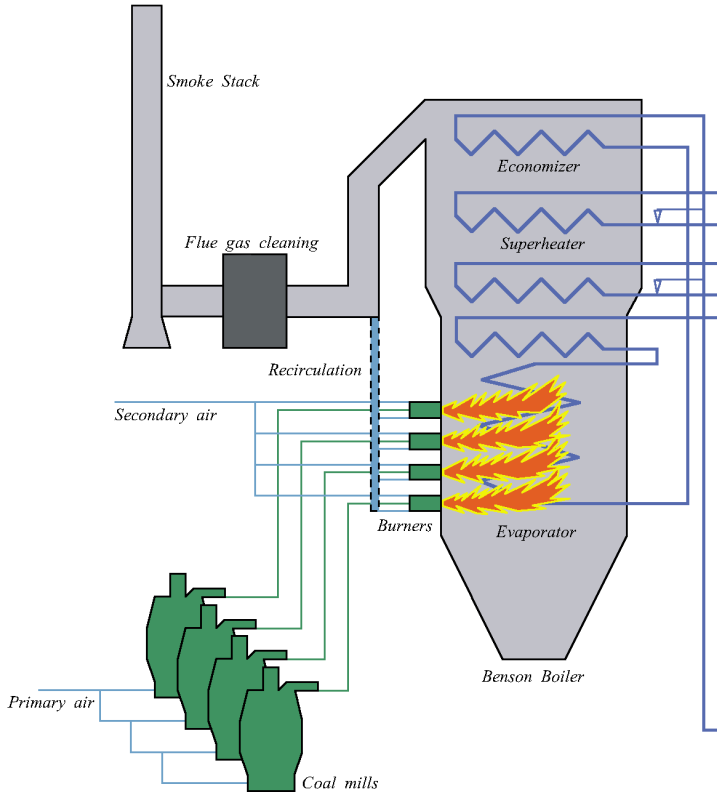


Figure 1.2: Illustration of the power plant considered in [Andersen et al., 2005].

The operation of power plants is described in [Joergensen et al., 2006], where a hierarchy of the control structure for a power plant is presented (see Figure 1.3).

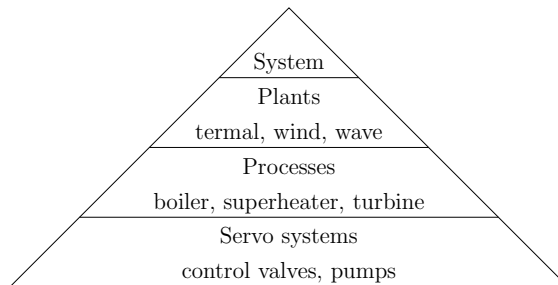


Figure 1.3: Illustration of the multiple levels in a power plant control system.

The top level is the complete system which is divided into different plants in the second level and processes in the third level. Finally in the fourth level is the individual servo systems consisting of e.g. pumps and control valves. A method for performing optimiza-

tion of performance in this hierarchy is presented and relies on economic steady-state optimization on system level. It is, however, concluded that the top level optimization depends on the low level.

In production economics the all possible outputs from a production unit or “firm” are identified and called the production set [Mas-Colell et al., 1995, Chapter 5]. The production units are seen as black boxes which are capable of transforming some goods (input) to other goods (output). Some assumption are often made about the production set e.g. *No free lunch* and *Free disposal*, i.e. the production set, Y , cannot contain \mathbb{R}_+^l as this would yield production of some quantity without consumption and the company can absorb any additional input without reducing the output. In [Mas-Colell et al., 1995, Chapter 5] it is concluded that the objective of a company is to maximize its profit, which at first seems reasonable. However, it is possible to imagine companies which have the objective of maximizing sales revenue or the size of the company, but if the company is owned by the consumers in a market they will agree that profit maximization is preferable regardless of their own preference function.

In the case of power plants there exists a Nordic market place, Nord Pool, where power contracts are negotiated [Nord Pool, 2009]. Here the price of electricity, as known by the average electricity consumer, is established as well as other quantities relating to the quality of the power deliverance. These quantities are traded on what is called the elspot market. An example of the quality of the power is when a power plant delivers more or less electricity to the grid than agreed. This is a problem as the demand and supply of electricity should always balance and therefore there is also a market for trading regulating power which is used for this balancing. The regulating power has two prices, an up price and a down price, which are used in accordance with what the electricity producers do, i.e. produce more or less electricity than previously agreed.

The data from Nord Pool has been used to schedule the usage of hydro power plant in Norway such that the production plan commitment of the current day is fulfilled while maximizing the profit of the hydro plant [Fleten and Kristoffersen, 2008].

1.2.4 Optimization

In this section three different optimization problems relevant for this work will be presented along with solution methods. The three considered problems are static optimization, dynamic optimization, and optimal control, which the following is examples of. Find $x \in \mathbb{R}$ that minimize

$$f_1(x) = x^2 + 1, \quad x \in [-1, 1], \quad (1.1)$$

find a continuous curve $x : [a, b] \rightarrow \mathbb{R}$ that maximizes

$$\int_a^b f_2(x, t) dt, \quad f_2(x, t) = -x^2 t, \quad x \in [-1, 1], \quad x(a) = x_a, \quad (1.2)$$

and find a control - a measurable function: $[a, b] \rightarrow \mathbb{R}$ that maximizes

$$\int_a^b f_3(x, u) dt, \quad f_3(x, u) = (x - u), \quad u \in [-1, 1], \quad x(a) = x_a \quad (1.3)$$

subject to $\dot{x} = -x + u$

The above problems are example of static, dynamic, and optimal control problems, respectively.

The cost function in these three different problems exist in many different forms depending on the optimization problem². In this work, it is assumed that optimization means maximization as a minimization problem can be converted to an equivalent maximization problem, e.g. the problem in (1.1) is equivalent to

$$-\max_x \left(-(x^2 + 1) \right), \quad x \in [-1, 1].$$

The argument is, however, the same, i.e., $x = 0$.

Static and Dynamic Optimization

In this section solutions to the problems stated above will be presented. It will be shown that an approximated solution to the later problem is possible by means of static optimization. First, Fermat rule is presented, which can be applied directly to the static optimization problem. Maximum/minimum of a function is, in general, obtained at stationary points, end points, or points where the differential does not exist.

Fermat's Rule Let $f : (a, b) \rightarrow \mathbb{R}$ be a function and suppose that $x_0 \in (a, b)$ is a local extremum of f . If f is differentiable at x_0 then $f'(x_0) = 0$.

By applying Fermat's rule to (1.1) it is possible to obtain solution candidate to the minimization problem, i.e.,

$$f'_1(x) = \frac{d(x^2 + 1)}{dx} = 2x,$$

which yields the solution $x = 0$ as a stationary point (and also the point of minimum in this case). Thus the minimum of (1.1) is 1. A similar approach can be used to solve the problem in (1.2) when it is realized that it is necessary to maximize $f_2(x, s)$ for each $s \in [a, b]$, i.e.,

$$f'_2(x) = \frac{d(-x^2 t)}{dx} = 2xt,$$

which yields the solution $x = 0$ for $t > 0$ as a stationary point (and also the point of maximum in this case³). Thus the maximum of (1.2) is 0. This kind of problem can also be solved using calculus of variation and the Euler-Lagrange equation which will be explained later.

The last problem is actually an optimal control problem which is the subject of the next section but it is possible to obtain an approximation of the solution, given some

²These could be linear or nonlinear. However, most results from linear problems can be extended to convex problems as well. Furthermore, the final time in integral could, in the general case, also be infinite time. This will, however, not be considered in this work.

³If (1.2) was minimized instead of maximized $x = 0$ would still be a stationary point, but one of the end point of x ($x = 1$ or $x = -1$) would yield minimum.

assumptions, by using static optimization. The assumptions needed is that control, u , is piecewise constant (in time), i.e.,

$$u(t) = u_k, \quad kh \leq t < (k+1)h,$$

for each time step, k , and where h is the sample time. Using this assumption and lifting (see [Chen and Francis, 1995]) it is possible to convert the problem to discrete time, i.e.,

$$\int_a^b f_3(x, u) dt \approx \sum_{k=0}^{N-1} cx_k + du_k,$$

where c and d are constants. The problem in 1.3 can now be rewritten as a static optimization problem given as

$$\max_{\bar{u}} D\bar{u} + C,$$

where \bar{u} is a vector of u_k 's by using the fact that the discrete system equations for the can be written as

$$\bar{x} = \bar{\Phi}x_0 + \bar{\Gamma}\bar{u},$$

with

$$\bar{\Phi} = \begin{bmatrix} 1 \\ \Phi \\ \Phi^2 \\ \vdots \\ \Phi^{N-2} \end{bmatrix}, \quad \bar{\Gamma} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \Gamma & 0 & \cdots & 0 \\ \Phi\Gamma & \Gamma & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \Phi^{N-3}\Gamma & \Phi^{N-4}\Gamma & \cdots & \Gamma \end{bmatrix},$$

and Φ and Γ are the discrete time system matrices. Thus now the problem can be solved using the methods above.

Optimization problems as presented in this section can, as long they are convex problems, be modeled (and solved) using a optimization tool for Matlab called yalmip [Löfberg, 2004].

Optimal Control

Optimization problems with dynamic constraints have been around for many years and different techniques for formulating and solving them have formed. In this section three of the techniques will be introduced starting with the oldest which is calculus of variation. Thereafter dynamic programming and maximum principle will be introduced.

A classical problem of calculus of variation is the Brachistochrone problem which considers a particle sliding (frictionless) along a curve, $x(t)$, between to points where the objective is to find a curve such that minimum time is obtained [Cesari, 1983]. This problem was solved by John Bernoulli in 1696. A classical calculus of variation problem, such as Brachistochrone problem, is formulated as [Vinter, 2000]

$$\min_{x(t)} \int_a^b L(t, x(t), \dot{x}(t)) dt.$$

A necessary condition for problems as above was found by Euler in 1744 [Seierstad and Sydsæter, 2007] and is often referred to as the Euler-Lagrange equation, which states

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x},$$

where $x(a) = x_a$ and $x(b) = x_b$ are known and fixed.

In the 1950s during the space race refinements of the calculus of variation developed for e.g. trajectory planning of rockets. The refinements were dynamic programming (Hamilton-Jacobi(-Bellman) equation) developed by Bellman [Bellman, 2003] in the United States and Pontryagin's maximum principle developed by Pontryagin [Pontryagin et al., 1965] in the Soviet Union. These methods were able to handle a larger variety of problems including problems of finding an optimal control input and thus these methods, among others, are referred to as optimal control. Problems defined as optimal control problems can often be reformulated as a problem of calculus of variation and vice versa. However, often the choice of solution method/problem formulation depends on the nature of the problem, customs, and what smoothness assumptions are imposed [Clarke, 1990]. In their classical forms the maximum principle is less restrictive than calculus of variation and dynamic programming.

Now, a closer look at dynamical programming and in particular maximum principle, which both have advantages and disadvantages. As an example following problem is considered. The objective is to control a system

$$\dot{x} = f(x, u), \quad x(0) = x_0, \quad x \in E \quad u \in U \subset \mathbb{R}^m,$$

where E is a open subset of \mathbb{R}^n and such that the cost

$$J_T(x, u) = \int_0^T g(x, u) dt + G(x(T)) \quad (1.4)$$

is minimized [Zabczyk, 2008]. This formulation is often referred to as a Bolza problem of optimal control.

Dynamic programming Assume that a real function $W(\cdot, \cdot)$, defined and continuous on $[0, T] \times E$, is of class C^1 on $(0, T) \times E$ and satisfies the equation

$$\frac{\partial W}{\partial t} = \inf_{u \in U} (g(x, u) + (W_x(t, x) | f(x, u))), \quad (t, x) \in (0, T) \times E,$$

where $(\cdot | \cdot)$ denotes the scalar product and with the boundary condition

$$W(0, x) = G(x), \quad x \in E.$$

Then

$$J_T(x, u^*) = W(T, x)$$

is the optimal cost and

$$u^* = k(t, x) = \arg \inf_{u \in U} (g(x, u) + (W_x(t, x) | f(x, u)))$$

is the optimal control input (see e.g. [Zabcyk, 2008] and [Jönsson et al., 2009]). Some properties of dynamic programming is that it gives sufficient condition and a feedback solution ($k(x)$) but it is necessary to solve nonlinear partial differential equations and the cost function needs to be sufficiently smooth [Jönsson et al., 2009].

Maximum Principle Maximum principle, on the other hand, can be used even when the cost function is not sufficiently⁴ smooth and usually it is easier to find an optimal solution than solve partial differential equations as will be demonstrated below but maximum principle only gives necessary condition [Jönsson et al., 2009]. The formulation of the maximum principle which will be given here is for optimal control problems of the Lagrange type, i.e., the cost function is given by

$$J(x, u) = \int_0^T h(x, u) dt.$$

It is possible to convert the Bolza optimal control problem in (1.4) to a Lagrange type, which is given by

$$J_T(x, u) = \int_0^T [g(x, u) + x^0] dt, \quad \dot{x}^0 = 0, \quad x^0(0) = \frac{G(x(T))}{T},$$

[Cesari, 1983]. Maximum principle for problems with fixed time interval, as the problem above, states that if a piecewise continuous control $u^*(t)$ defined on $[0, T]$ solves the problem and $x^*(t)$ is the associated optimal path. Then there exists a constant λ_0 and a continuous and piecewise continuously differentiable vector function $\lambda(t)$ such that for all $t \in [0, T]$

$$\begin{aligned} (\lambda_0, \lambda) &\neq (0, 0), \\ H(x^*(t), u^*(t), \lambda(t), t) &\geq H(x^*(t), u, \lambda(t), t) \quad \forall \quad u \in U, \end{aligned}$$

where

$$H(x, u, \lambda, t) = \lambda_0(g(x, u) + x^0) + \langle \lambda, f(x, u) \rangle.$$

Except at points of discontinuities of $u^*(t)$

$$\dot{\lambda}(t) = -\frac{\partial H(x^*(t), u^*(t), \lambda(t), t)}{\partial x}, \quad \lambda(T) = 0,$$

which is often referred to as the adjoint equation and furthermore $\lambda_0 = 1$ (see [Seierstad and Sydsæter, 2007] for further comments on the maximum principle as given here).

The problem of finding a simultaneous solution to the adjoint equation and system equation is referred to as the two-value boundary problem as the initial value is given for the system equations and the final value is given for the adjoint equation and is a hard problem to solve. Furthermore, numerical tools for solving an optimal control problem exist, e.g. DIDO or GPOPS (see [DIDO, 2009] and [GPOPS, 2009] for further details) which both can handle the Bolza type of problems. They deliver an approximation of the optimal open loop input and corresponding optimal state trajectory.

⁴Well, at least not as sufficiently as dynamic programming.

1.3 Outline of the Thesis

This thesis is a collection of publications and it is divided into two parts; an introduction and overview of the contribution and the contributions themselves. As the observant reader might have noticed part one has already been begun with an introduction and state of the art in Chapter 1. The approach taken in this work will be described in Chapter 2 where the contribution will be explained also. Part one closes with some suggestions for future work and conclusions on the this work in Chapter 3.

Four of the publications made during this PhD project are appended in part two in chronological order and are listed below. Furthermore, a note made concerning different approaches for formulating the problem is appended as a technical note.

In part two the publications enclosed in this PhD thesis are

Paper A [Kragelund et al., 2008] This is the first paper where the business objectives of Dong Energy in relation to selection between different fuels are formulated. Given a time the optimization in this paper was performed by searching through the input space and evaluating the result of all possibilities.

Paper B [Kragelund et al., 2010] This paper is an extension of another paper writing during this PhD-thesis for European Control Conference [Kragelund et al., 2009b]. The problem is in Paper B described in a more formal manner and the solution technique is more mathematical funded. Real measurement data is used in the optimization and a discussion is made about the correctness of the price data and reference data. Furthermore, a proof of existence of solution to the problem with dynamics is also included in this paper.

Paper C [Kragelund et al., 2009a] In this paper the dynamics of the fuel systems are included in the problem formulation. The optimization is performed in discrete time under some assumption of sample-hold and piecewise constant approximation of the time data.

Paper D [Kragelund et al., 2009c] In this paper some of the constraints are reformulated due to some observations made in Technical Note (see below). Pontryagin's maximum principle is then applied to the problem and an optimal input strategy is developed which is a combined feedback and feedforward.

Technical Note In this note different approaches for formulation the optimization problem has been examined and compared. The solutions to the different approaches have been developed in discrete time.

2 | Summary of Contributions

In this chapter the problem is formulated and the relation between the different papers in part two will be explained as well as the ideas behind the different approaches.

2.1 Power Plan Operations

In this work, the focus has been on the fact that the ultimate goal of a company is to maximize profit and therefore, a monetary optimization functional, which is price, has been put on the objectives of the company. This also has the advantage that the different objectives (with different units) are mentally easy to assess against each other, i.e., the designer should try to fulfill the (those) objective(s) that yields the highest profit. In this work the considered company has been an imaginary¹, DONG Energy operated, power plant capable of using three different fuel systems. The fuel systems considered in this work consist of a coal system, a gas system, and an oil system. Each of the fuel systems has certain advantages and disadvantages. Coal is an inexpensive fuel but using coal imposes some restrictions on how fast it is allowed to change the production as the coal is first grinded in coal mills before the dust is burned in the furnace which makes it difficult to control the production precisely. When using gas or oil, on the other hand, it is allowed to change the production faster as the fuel flow can be measured directly and controlled with a simple actuator which makes it easy to control the production. Gas and oil are, however, expensive fuels and thus for a given production/demand of electricity they will deliver a lower profit (a more detailed description of the different fuels and the advantages can be found in Paper A and B - see page 39 and page 55). In this work we assume that the dynamics of the different fuel systems are decoupled, i.e., the concatenated dynamics yields a block diagonal system matrix. Furthermore, the model structure of each fuel system is assumed as a third order linear system as in [Edlund et al., 2009b] and the time constants used are 60s, 70s, and 90s for gas, oil, and coal respectively. In this work it is assumed that the boiler model is a static mapping from fuel flow to the objective measures, which are explained next.

DONG Energy has four different business objectives, which deals with Controllability, Efficiency, Availability, and Life Time. In this work the two first objectives are interpreted in terms of a boiler of different fuel systems, i.e., the functions, which in some manner describes efficiency and controllability is formed.

¹Imaginary in the sense that a power plant capable of using the three considered fuels does not presently exist. We do, however, consider it a DONG Energy plant as their business objectives have been used.

Efficiency is a measure of how efficient different aspects of the company is, e.g., the boiler, the turbine, and management. In this work this objective is interpreted as how much electricity is produced given an amount of fuel, i.e., the conversion of fuel flow into electricity.

Controllability is a measure of how well the production can be controlled or fast it is possible to change production. In this work this is interpreted as how much the electricity production can be changed given a operating condition.

Availability is a measure of how much of the time the company is able to operate without break down. It could also be a measure of much the production could be increased.

Life Time is a measure of how long different components in the company can be used before they exist to work or are used up.

Each of these objectives has been established as maximization of either of them leads to a greater profit of the company, e.g. if the life time of a power plant is prolonged or if the produced electricity given an amount of fuel is larger then the profit will over time be larger than if these life time was short or less electricity was produced.

The value of these objectives have been established using the price data available at Nord Pool [Nord Pool, 2009] and in collaboration with DONG Energy by using their heuristics. E.g the current and historic prices of electricity is available online and are depicted in Figure 2.1 over a 30 day period (Marts 28th 2009 to April 26th 2009).

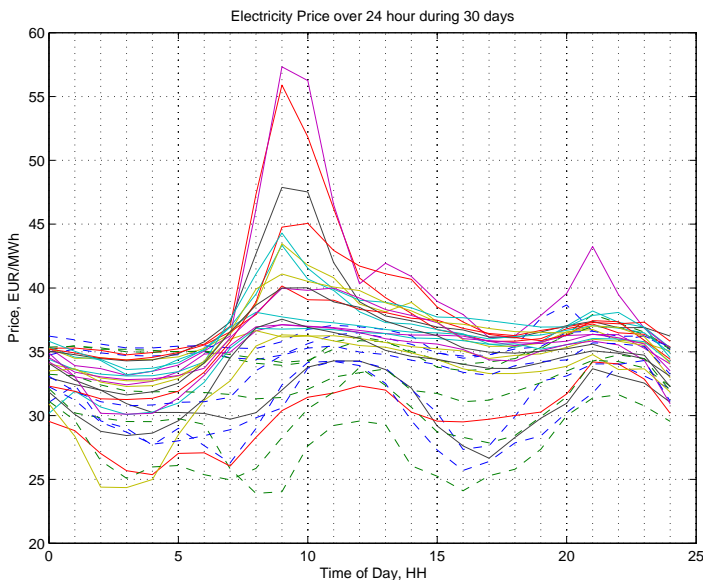


Figure 2.1: The efficiency price over 24 hours from Marts 28th to April 26th 2009, where each day is depicted by a new graph. The dashed graphs illustrates the data for weekends and solids the weekdays. The data used to generate this plot has been found on www.nordpool.dk

The high level control and planning structure of a power plant is illustrated in Fig-

ure 2.2 where black illustrates the current configuration and blue indicates the additions proposed by this work (note that the $y_p^* = y_p$ when the current configuration is considered). A production plan, y_p , is provided to the power plant with the next 24 hours of

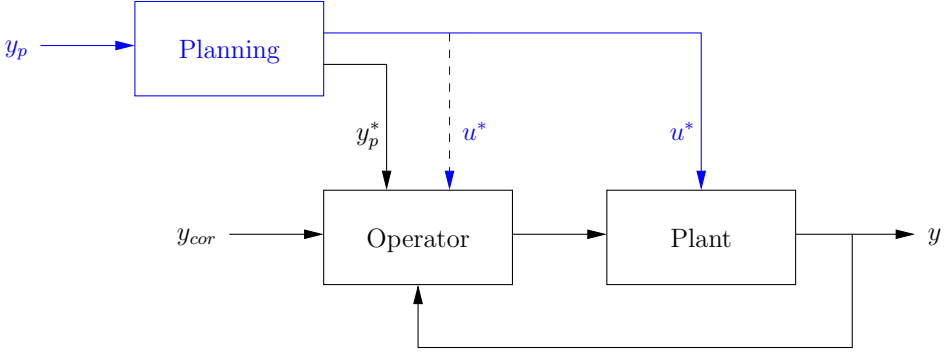


Figure 2.2: Illustration of the high level control and planning structure of a power plant. The blue color indicates the addition proposed by this work and $y_p^* = y_p$ when the blue additions are omitted.

operation and traditionally, this production plan is delivered to the operator who controls the plant. The operator then controls the fuel flow into the plant such that the electricity prescribed by the production plan is generated. As the predictions used to generate the production plan not necessarily fit the real life demands exactly, a correction is needed which is delivered by the electrical grid responsible. This correction signal is fed to the operator, which adjusts the plant production accordingly (for further details see [Edlund et al., 2009a], [Joergensen et al., 2006], and [Edlund et al., 2008]).

A prognosis of the next day's electricity consumption is established by Energinet.dk [Energinet.dk, 2009] which is responsible for the electrical grid in Denmark. The estimated electricity consumption in an area (e.g. West Denmark) is divided between the different electricity producers in accordance with the bids on Nord Pool and thus a production plan is generated for each producer. The production plan used in this work has been delivered by DONG Energy and is depicted in Figure 2.3. During the night the production is low and then between 6:00 and 7:00 the expected production rises and stays high during the day as seen in the figure. In the afternoon and evening the production fluctuates as production companies shut down and people return home from work.

In this work we introduce an additional planning level before the operator in the operation of the power plant (see Figure 2.2). The planning level selects an optimal actuator configuration for the particular production plan according to the business objectives (this is illustrated by the “Planning” block in Figure 2.2). The planning consists of selecting which actuator systems should be used and a reference for these actuator systems is delivered to the operator.

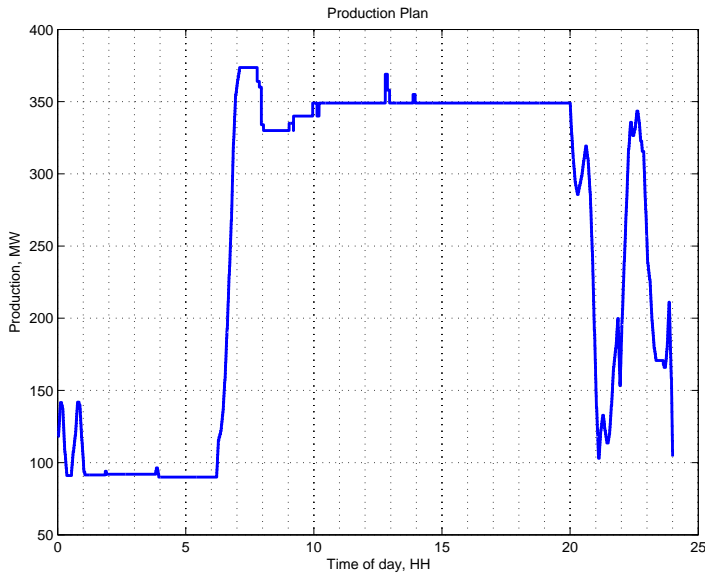


Figure 2.3: The production during June 29th, 2008. The data used to generate this plot has been provided by DONG Energy.

2.2 Relation Between the Contributions

In the following the different papers of this work will be presented along with the relation between them. The difference between them will be illustrated by block diagrams, showing where the differences arise. The papers have been divided into two categories, which are static optimization and dynamic optimization, where static is meant as the dynamics of the fuel systems are not considered where as the dynamic approaches do consider these dynamics.

2.2.1 Static Optimization Approaches

In this section the first two papers from part two will be presented in chronological order, i.e., Paper A entitled “On Propagating Requirements and Selecting Fuels for a Benson Boiler” and Paper B entitled “Optimal Usage of Coal, Gas, and Oil in a Power Plant” are presented.

Paper A

The primary result of Paper A (see page 39) is a manual hierarchical breakdown of the considered power plant and the development of models of the different business objectives of DONG Energy as function of the different fuel systems. The model of the plant considered in Paper A was a black box model as illustrated in Figure 2.4, where the three inputs are the fuel flow of each of the fuel systems and the output from the black box is the sum of the company’s income, $\frac{dkk}{h}$, for the objectives controllability, efficiency,

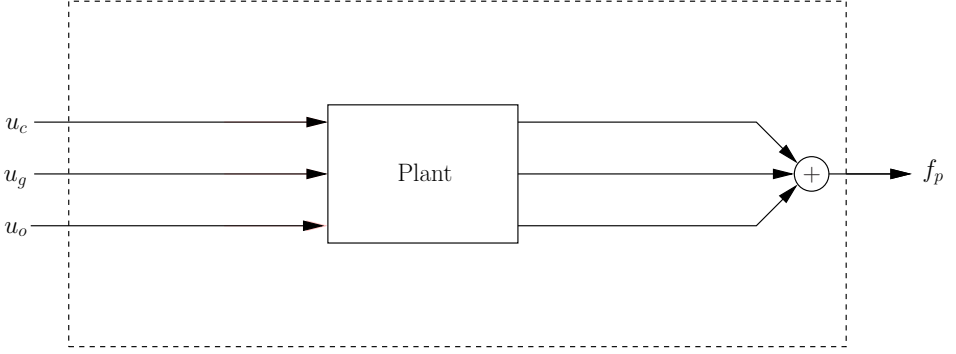


Figure 2.4: Illustration of the plant model used in Paper A, which is a black box model with three inputs and one output.

and availability. The price of each of the objectives and the fuels was considered as constants and included as part of the complete model. The optimization of the total income was performed by searching through the different configuration for the solution which yields the greatest income in $\frac{dkk}{h}$ given a certain production load. For each configuration YALMIP [Löfberg, 2004] was used to formulate and solve the problem. This paper should only be thought of as an introduction to the problem considered in this work. The presented mathematical formulation of the problem and solution are not completely technical sound, but should be considered as best effort at the time of publication. For specific details the reader is referred to the note at the end of the paper and for more rigorous mathematical formulation the reader is referred to the later work presented below.

Paper B(a)

As the model in Paper A does not reveal much information about how the prices of the different objective influence the plant profit a notion from production economics is employed in [Kragelund et al., 2009b] (this paper will be denoted Paper B(a) in the following). In this paper we separate the price on each objective from a measure (or output) of these objectives. Paper B(a) has not been included in part two of this thesis as many of the ideas also are presented in Paper B, however there exist some differences between paper B and Paper B(a) and they will be explained here as well (see Figure 2.5). As revealed by comparing Figure 2.5 with Figure 2.4 the model of the plant now includes information of the price data, where the measure of the objectives are the outputs of the plant which is fed through a model of the value of each of the objectives and thus obtaining the revenue of the company. Furthermore, the “ p_C ” box calculates the expenses of use the fuel which is subtracted from the earnings and the profit is obtained. The parameters in p_E are time varying according to the demands of market as described by Nord Pool. However, in this paper piecewise affine functions are used as approximations of the time-varying price data and production plan delivered to the plant, i.e., the real market data is approximated by nice functions in Paper B(a).

The optimization is performed using methods from static optimization and taking advantage of the fact that the optimal input is a vertex of a 2-simplex in \mathbb{R}^3 for a given

production set point. When a time varying production reference was introduced it was

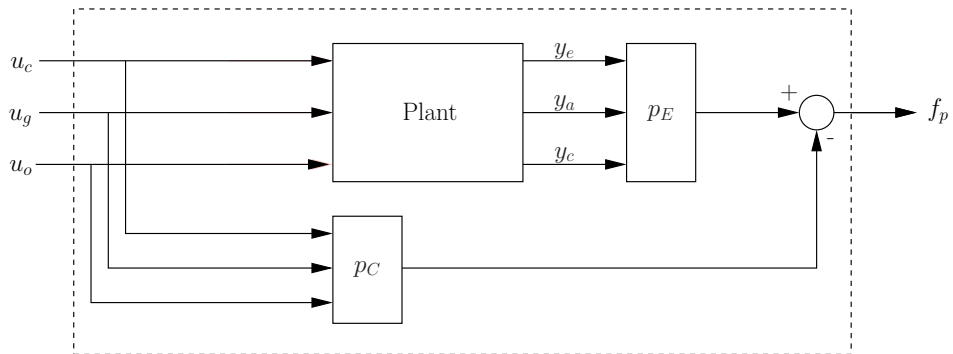


Figure 2.5: Illustration of the plant model used in [Kragelund et al., 2009b]. During the development of this model notions from production economics have been used and the price data has been separated from the objective measure.

found that the optimization procedure carried out for the fixed production set point could be used.

It was concluded that the result of paper B(a) could be used online to determine which fuels to use during the day as described in Section 2.1 but also offline to determine if a plant should be instrumented with additional fuels installations and equipment.

Paper B

In Paper B (see page 55) the same line of thinking as in [Kragelund et al., 2009b] is used. However as seen in Figure 2.6 the block diagram has changed a bit, i.e., the availability objective is no longer considered. The reason for removing the availability is that the results in [Kragelund et al., 2009b] reveals that this measure does not add any additional information to the optimization, which is due to the method the availability is modeled. The availability is modeled as $y_a = C - \sum y_e$, where C is a constant and $\sum y_e = y_p$, which is the given production reference, i.e., the availability is determined by the production reference which is given. Furthermore, some trends in the production reference and price data are established by examining the measurement data over a 30 day period and it is shown that June 29th 2008 is a typical day. Therefore, the production reference and price data in Paper B is the real data for June 29th, 2008.

The same method for optimization as in [Kragelund et al., 2009b] is used however additional constraints on how large fuel flow each of the fuel systems are capable of delivering is imposed in the optimization, i.e., an upper limit on the produced electricity by each system is included. Besides optimizing the profit of the power plant there is also given an existence result on the optimal solution in presence of fuel system dynamics.

When the result from this paper is compared to a case where only coal is present an increase in the profit by 12% or 8% over 24 hours of operation is obtained depending on which constraint is imposed on the fuel flows.

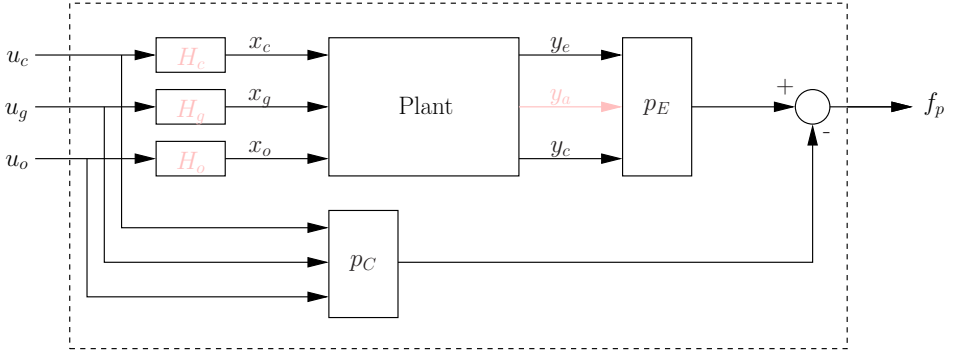


Figure 2.6: Illustration of the plant model used in Paper B, where the availability objective has been omitted and measurement data has been used in the optimization.

2.2.2 Dynamic Optimization Approaches

Both Paper C and Paper D (see page 79 and page 95) consider the problem when fuel system dynamics is included. A block diagram of the model considered in these papers is depicted in Figure 2.7.

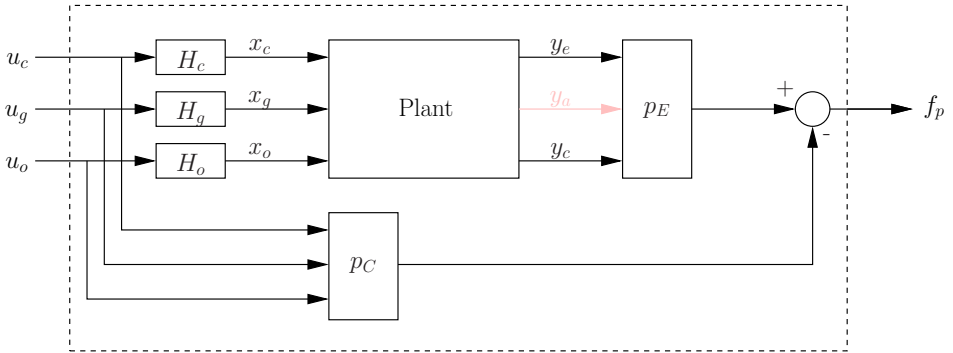


Figure 2.7: Illustration of the model used in Paper C and Paper D, where the dynamics of the fuel systems are considered.

As seen in the figure the input for the fuel flow enters a dynamics block containing dynamics which in this work is 3rd order linear time-invariant. To reuse the established objective models the dynamics of production using the different fuel system are assumed to be completely decoupled from the boiler and other components necessary for power production. Otherwise, the rest of the model is assumed as in the papers A and B.

Paper C

In Paper C the profit function is discretized² and optimization is performed in discrete time using linear programming, i.e., the method described used to solve the problem in (1.3) in Section 1.2.4 is used to convert an optimal control problem into a discrete time linear program. The production reference is implemented as a side-constraint in the optimization such that the electricity production is within a margin of α [MW] to the reference.

Comparing the results of this work with the results from the static optimization approaches described previously it was concluded that the dynamics of the fuel systems does influence the profit of the company but the fuel usage during the day is comparable and the profit from mixing fuels are larger than the case of only using coal. Furthermore, it is concluded that a full³ gas and oil system is not necessary as they are only partially used. The final result in Paper C is an open loop control signal which optimizes the profit of the company.

Technical Note

As indicated above it is possible to solve the discrete time linear program, however, it is time consuming and therefore different alternative approaches for reformulating the problem to obtain a less computational heavy solution are considered. These different approaches are described in Technical Note (see page 125) and they include less frequent sampling of the tracking side constraint and inclusion of the tracking requirement in the cost function as a penalty in the quadratic tracking error. The less frequent sampling of the tracking side constraint yielded large fluctuations in the tracking error and therefore an additional constraint on the allowed changes of the input was imposed to remedy this. It is, however, concluded in the Technical Note that the best solution for lowering the solve time is to include the tracking requirement in the cost function as a penalty on the quadratic tracking error as this yields the best solve time compared to the tracking of the reference.

Paper D

In Paper D the approach of including the reference tracking in the cost function is taken, however the problem is formulated in continuous time. By using maximum principle an optimal control strategy is obtained, which does, however, depend on an unknown switching function. To approximate this switching function the problem is converted to discrete time and the switching instances are obtained. The developed strategy provides a continuous time feedback solution which is much more robust towards input noise than the discrete time solution. The developed control strategy yields a 30% larger profit than what was possible by using the discrete time input strategy, i.e., the reference tracking is better.

²The profit function is converted into an discrete time equivalence by assuming the time varying function can be approximated by piecewise constant functions.

³Full means in this regard a system capable of delivering fuel for full production of the plant.

2.3 Discussion About the Developed Approaches

In this section a discussion about the different approaches presented above will be given. Then some results on a problem similar to the one considered in part two will be given, i.e., a power plant capable of using coal and oil will be discussed.

The three approaches above have different advantages and disadvantages and e.g. the first methods are less computational but it is also believed that they are less accurate than the latter. Furthermore, the later results does not only tell when to use different actuator systems but also how they should be used to obtain the optimal profit. The first approaches are, however, relevant as they can give a hint of a possible benefit from using multiple fuels and it is also here the idea of profit maximization and models for the later work is developed.

Online Production Reference Correction

In Paper D the control signal for the power plant is a combined feedforward and a state feedback (see (7.11) page 103), where the time-varying feedforward (C_t in (7.11)) depends on the production reference and the market price data and their derivatives. When production reference corrections are introduced the derivatives need to be calculated, however, if the reference changes are known a certain amount of time in advance it is possible to approximate the derivatives. Furthermore, the largest component of C_t is the reference and it is expected that the production corrections are small, therefore it might be possible to simply ignore the derivatives of the changes in the feedforward signal.

2.3.1 Coal Fired Power Plant

A power plant capable of using coal and oil is considered in this section. This is an relevant plant to consider as most existing coal fired power plant fall under this category, i.e., a secondary fuel system is needed in a coal fired power plant during start of the system. This case has been considered using the methods from Paper C, however the other methods show similar results. The models are as described in these papers with the exception that the gas system has been removed by omitting its contribution in the profit function.

Linear Optimization with Fuel System Dynamics

The discrete linear optimization method from Paper C is in this section applied to a plant using coal and oil, i.e., the following linear program is solved and a optimal input sequence is found.

$$\max_{\bar{u} \in \bar{U}} V\bar{u} + C,$$

where \bar{u} is a vector of the inputs at each sample time and V and C makes up the sampled counterpart of the profit function developed in Paper C. The optimal input sequence vs. time is depicted in Figure 2.8 along with the profit vs. time. The oil system is used slightly during periods where the production reference changes rapidly as seen in the figure and at the end of the day the final profit of the company is 308000 *dkk*. When comparing this result to a plant using only coal, the profit is actually lower. The profit of a power plant

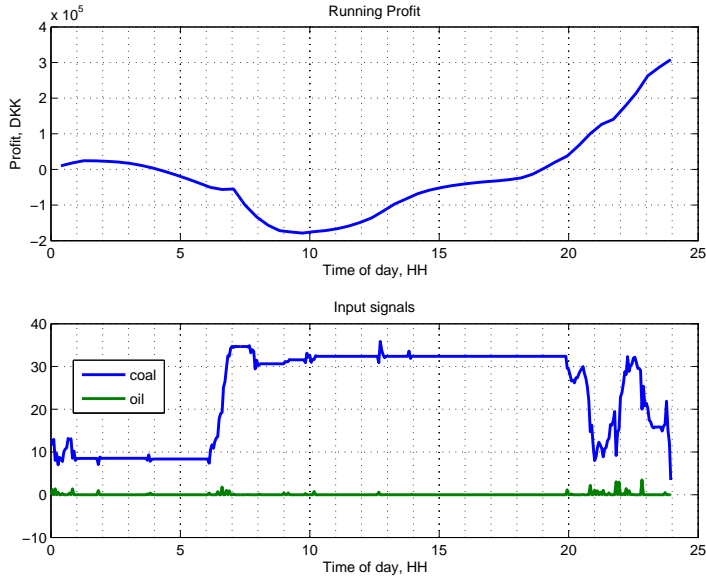


Figure 2.8: Optimization results for a plant capable of using coal and oil. The final profit of the company is at the end of the day 308000 *ddk*.

capable of using only coal is depicted in Figure 2.9 and as seen in the figure the profit is slightly larger at the end of the day.

Both the coal only plant and the plant capable of using oil and coal have a larger tracking error than the plant considered in Paper C and furthermore, the profit of the plant in paper C is larger. Thus a plant capable of using gas is beneficial and the above suggests that current coal plants should have been instrumented with a secondary gas fuel system instead of an oil system.

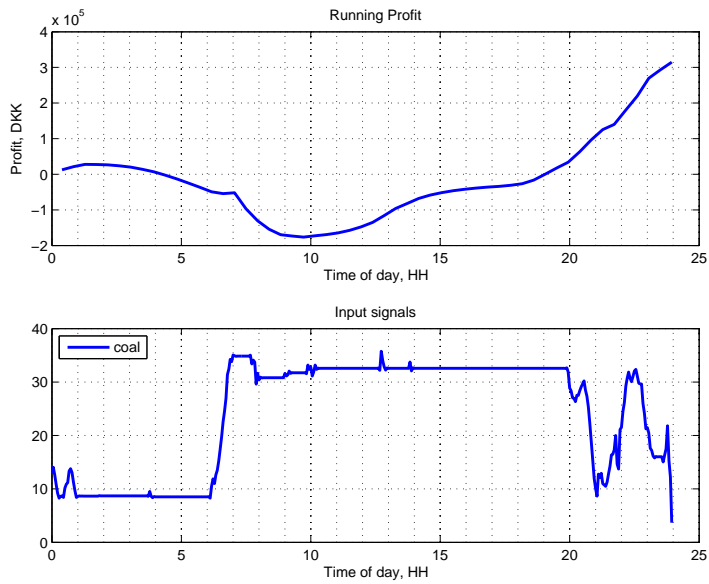


Figure 2.9: Optimization results for a plant using only a coal fuel system. The final profit is 315000 *dkk* at the end of the day.

3 | Concluding Remarks

In this chapter some concluding remarks about the work in this thesis will be given. First, some suggestions to future work and notes on the perspectives will be given and last conclusions will be drawn.

3.1 Future work and Perspectives

Some suggestions for future work will be presented in this section. Some of the suggestions arise from the different papers which have not been pursued further by this work while others are of a more general nature.

- This work assumes that the business objectives of a company can be propagated to the actuators of the plant such that profit maximization of the jointed economic value of the business objectives leads to a usage plan for each of the actuators. However, it would be relevant to develop methods (or a procedure) to automatic propagate the business objective to the instrumental level of the plant.
- In this work simple models of two of the business objectives have been formulated but more detailed models might reveal additional benefits and more precise estimates of economical gain from using additional fuels - especially modeling of the remaining business objectives could be relevant.
- The market models could be expanded by e.g. using predictions of prices and demand further into the future and in particular the model of the controllability price could be improved. Furthermore, models of the Nord Pool bidding system and settling of the price could be incorporated as well as environmental perspectives such as CO_2 emission, which has gained more focus in recent years.
- With the recent focus on environmental friendly energy production the current electric market is going to change during the next couple of years as more of these renewable energy systems are incorporated into the power grid. To ensure integrity of the electrical grid with the (rapid) varying production from these renewable energy system, it is expected that new “actuator” systems will become available e.g. decentral short-time storage of energy in form of electrical car (see [Edison Net, 2009] and [Better Place, 2009]). An obvious extension of the methods in this work is to incorporate these new “actuator” systems in the optimization.

- As a consequence of the above constraints involving the electrical grid-capabilities are necessary. However, including such constraints could also be interesting together with prediction of the wind, i.e., if models of the electricity production from wind energy is included in the optimization it would be possible to perform better planning of the electricity production on thermal power plants.
- The above suggests optimization on multiple power plants and coordination of production. The distribution of electricity production between different power plant within the same company has been studied by [Edlund et al., 2009a] where the coordination between DONG Energy's power plant in Western Jutland is considered (automatic generation control). A scheme for a stable coordination controller is proposed where participation factors are used to distribute the total production between the different plants.
- As noted earlier, new electricity producing devices (e.g. renewable energy system) are added to the grid over time, which suggests that online addition of new devices could be considered to allow for plug and play of these devices.
- As there are discontinuous switches in the profit function the problem described in this thesis could be an obvious candidate for nonsmooth analysis (see [Clarke, 1990]). Techniques from nonsmooth analysis have been applied to optimal control and maximum principle [Vinter, 2000] and therefore, it would be quite straight forward to formulate the problem using nonsmooth functions. Applying nonsmooth analysis and a nonsmooth version of maximum principle, on the other hand, would require more work but it is believed that this would be an interesting subject for further investigation.
- Last suggestion for future work deals with the feedforward in Paper D, which consists mainly of the production reference signal (and derivatives as noted earlier) and it might be possible to utilize this when correction in the production reference is provided by the electrical grid operator.

3.2 Conclusion

Three approaches for developing a control strategy for maximizing the profit of a power plant have been presented. The first approach uses an black box model of the plant and optimization is preformed using search of the input space at a given time. The second approach utilizes a notion from production economics, which suggests dividing the price data from the objective measure. This enables changes of the price data over time, which can be used in the optimization. The third approach include the dynamics of the fuel systems in the optimization and methods from discrete optimization and optimal control are used.

This work has shown that, using the developed models of the business objectives, it is possible to enlarge the profit of a coal fired power plant with up to 30% during a day if the plant is capable of using an additional gas system.

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Contributions

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Paper A

On Propagating Requirements and Selecting Fuels for a Benson Boiler

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Abstract

In this paper, the problem of optimal choice of sensors and actuators is addressed. Given a functional encapsulating information of the desired performance and production economy the objective is to choose a control instrumentation from a given set to comply with its minimum. The objective of the work is twofold: reformulation of the business objectives into mathematical terms and providing solution to the given optimization. Commonly, there exist overall business objectives which dictate how a plant should be instrumented and operated either directly or indirectly. The work shows how to propagate a global objective to local subsystems. Particular focus is on a boiler in a power plant operated by DONG Energy - a danish energy supplier. The business objectives have been propagated to the actuator level to allow for selection of an actuator configuration.

1 Introduction

The selection of sensors and actuators has usually depended greatly on the designer's system knowledge, however, in recent years more focus has been made on developing tools to aid the designer during this phase as processes are becoming more complex and difficult to assess. One such tool is the Relative Gain Array, which is used to pair inputs and outputs in a multiple input multiple output system to enable decentralized single input single output control [1, page 90].

The placement of sensors and actuators has been studied for different applications and [2] reviews methods used in the aerospace industry. More general purpose methods for selecting and placing sensors and actuators have been evaluated in [3] and [4], which include e.g. methods relying on controllability measures such as state reachability and more sophisticated methods using robust performance measures. It is also concluded in [3] that the choice of sensors and actuators dictates the expenses for hardware, implementation, operation, and maintenance.

A software requirement specification procedure is presented in [5] which is used on an industrial aircraft collision avoidance system (TCAS II). They conclude that the model used during specification should resemble the real world to allow the designer to used his/her system knowledge.

The requirements for a process control system are specified for the very top level. They reflect cost, reliability, availability, survivability, and dependability. The aim of this work is to investigate how the selection and placement of sensors and actuators influence such measures and eventually how the measures influence the selection and placement of sensors and actuators.

1.1 Outline

This paper presents the first results gained from the case study of a power plant operated by DONG Energy. The objective is to gain an insight into what challenges arise when propagating business objectives to the selection of sensors and actuators. First, an introduction to the problem is given in Section 2 including a presentation of the plant used to illustrate the problem. Thereafter, our approach to propagate the objectives is presented in Section 3 along with some preliminary results on actuator selection for the

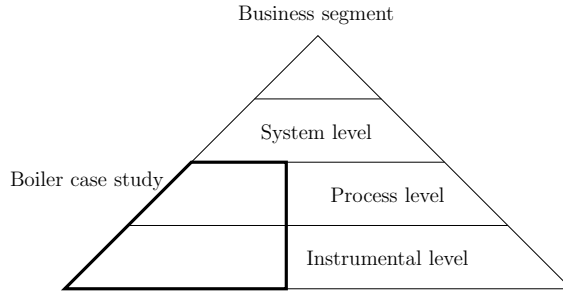


Figure 4.1: Business hierarchy showing the location of the boiler case study.

presented plant. Finally a discussion is made about the results and the future work within this program.

2 Problem Statement

The top level business objectives for DONG Energy deal with Efficiency, Availability, Controllability, and Life Time but the ultimate goal is to maximize DONG Energy's profit. In the collaboration with DONG Energy a coal fired boiler - a vital component of a power plant - is used in a test process as it possesses many of the aspects for propagating business level objectives to subsystem requirements and thus in selection of sensors and actuators. Figure 4.1 illustrates how the boiler is placed in an overall business hierarchy.

The model considered in this paper consists of the following components:

Coal mills The coal mills grind the coal to small dust particles which burn quickly and efficiently. However, it is difficult to control the amount of dust the coal mills deliver as it is not possible to measure the dust flow into the furnace.

Furnace The furnace is a module where the coal dust (or other fuels) is burned thereby delivering heat to the boiler.

Evaporator The evaporator is fed with water, which is evaporated under high pressure by the heat from the burners.

Superheater The superheater (super) heats the steam from the evaporator.

Economizer The economizer uses some of the remaining heat in the flue gas to preheat the feed water before it enters the evaporator.

The individual parts of the model are illustrated in Figure 4.2. However, the model does not consider the flue gas cleaning and smoke stack. Furthermore, the conversion from steam power to electrical power is also omitted but it is assumed that when running at full load the electrical power produced will amount to $400MW$.

To simplify this test process it is chosen to focus on the actuators in the system and the current model is added two additional fuels, which are gas and heavy oil. Some characteristics of the different fuels are:

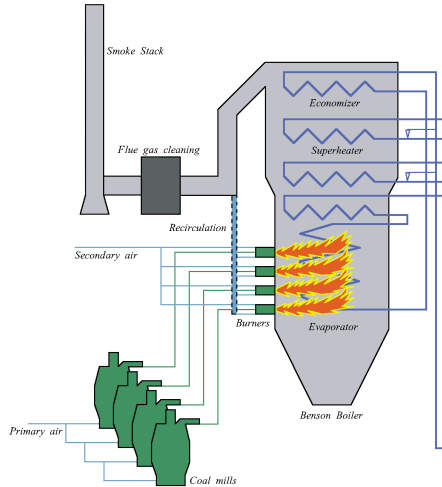


Figure 4.2: Benson boiler model.

Coal is advantageous when considering the price per Giga Joule (GJ) of stored energy, however, it is difficult to control as the nature of the coal mills introduces fluctuations in the coal flow, which are impossible to measure. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind and dry the coal which needs to be considered.

Gas arrives at the power plant under high pressure which is lowered using a turbine generating electrical energy. Furthermore, gas is more expensive than coal and energy within the gas is not converted to steam as efficient as with coal due to the layout of the chosen boiler. However, gas is much easier to control as it is possible to measure the flow.

Heavy oil is, with the current market prices, the more expensive of the three fuels but does have other advantages; it is possible to measure the oil flow into the boiler. However, it needs to be heated before entering the boiler and this requires energy placing oil between gas and coal when considering the own-consumption.

To get a better view of the different subsystems and their interaction the boiler model has been divided in a hierarchical manner depicted in Figure 4.3 (only the fuel part has been completed to actuator level). Using this breakdown of the boiler model it is possible to determine how to propagate requirements from boiler level to the individual actuators and ideally this propagation and selection would happen automatically¹.

¹In this paper the system knowledge of the DONG Energy collaborators is used.

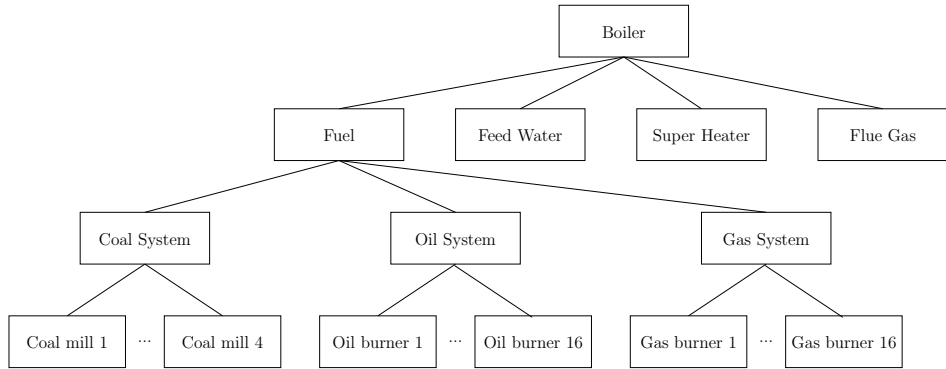


Figure 4.3: Example of how a boiler consists of multiple subsystems which can be used to propagate requirements from boiler level to the individual subsystems.

3 Performance Specification

In this paper the idea is to propagate the business objectives to the bottom of the hierarchy manually by setting up functions relating the objectives to the input and output of the system. If possible this task should with time be automatic or at least some framework aiding the designer in this task should be developed, however, in this paper a heuristic approach has been applied using DONG Energy's system knowledge. The functions should map to some monetary value of using the different fuels in relation to the business objectives and thus enable selection of an actuator configuration. Some of the parameters reflecting the different objectives change in time, e.g. the prices of the fuels and the demands of the electrical market. However, in this paper a certain market situation is considered and thus the problem becomes a static optimization problem. Furthermore, the functions set up is affine (or close to) and it is therefore chosen to use a linear programming framework to solve the optimization problem.

Three of the business level objectives - Efficiency, Availability, and Controllability - have been translated directly to the actuator level, i.e., simple functions describing the objectives in terms of the individual fuels have been established. Each fuel system comprises multiple sensors, actuators, and control loop, however, they are seen as individual actuators in this paper.

3.1 Efficiency Objective

Bearing in mind that the focus is the fuel system a high efficiency is desirable as less fuel will be needed yielding less expenses. Certainly, the expenses also depend on what kind of fuel is used as the market prices for gas, oil, and coal are not the same. Furthermore, the three fuels have different efficiency in converting the energy stored in the fuel into steam/electricity². The costs of preprocessing of the three fuels is also different as mention earlier.

²The different efficiencies are assumed to be caused by the manner the individual fuels burn

In this paper the income from production has been set to $200 \frac{dkk}{MWh}$ which was approximately what DONG Energy was paid when this study was established. The fuel prices have been set at $72 \frac{dkk}{MWh}$, $104 \frac{dkk}{MWh}$, and $180 \frac{dkk}{MWh}$ for coal, gas, and oil respectively (these prices were taken from a DONG Energy document). Furthermore, the preprocessing costs have been evaluated as constant loss or gain in energy. Each coal mill uses approximately $1MW$, however, the energy consumption is dependent on the load of the mill but in this paper the total consumption of the four mill is modelled as a constant loss of $4MW$. No data has been found on the energy consumption of the heater used for the oil but it is regarded as substantially lower than the coal mills and has therefore been set to $1MW$. Finally, the gas turbine used to lower the gas pressure generates $5MW$.

The efficiency has been found from measurement data from two power stations operated by DONG Energy and a function has been fitted to the measurement data for each fuel. The total expenses is calculated as total energy produced divided by the efficiency, i.e., the efficiency objective has been modelled as

$$J_e(\mathbf{x}) = 200 \frac{dkk}{MWh} \mathbf{x} - \begin{bmatrix} \frac{(x_{1:4} + 4MW)72 \frac{dkk}{MWh}}{0.00018x_{1:4} + 0.44} \\ \frac{(x_{5:20} - 5MW)104 \frac{dkk}{MWh}}{0.00031x_{5:20} + 0.37} \\ \frac{(x_{21:36} + 1MW)180 \frac{dkk}{MWh}}{0.00018x_{21:36} + 0.37} \end{bmatrix} \quad (4.1)$$

where \mathbf{x} is a vector with 36 entries containing the load in MW of four coal mills, 16 gas burners, and 16 oil burners respectively. Figure 4.4 depicts graphs of function J_e (when the cost of coal, gas, and oil is added individually) and as seen in coal is the only fuel yielding any income when only considering the efficiency objective. That is the price of gas and oil is too high when only considering the stored energy and discarding other benefits these fuels have.

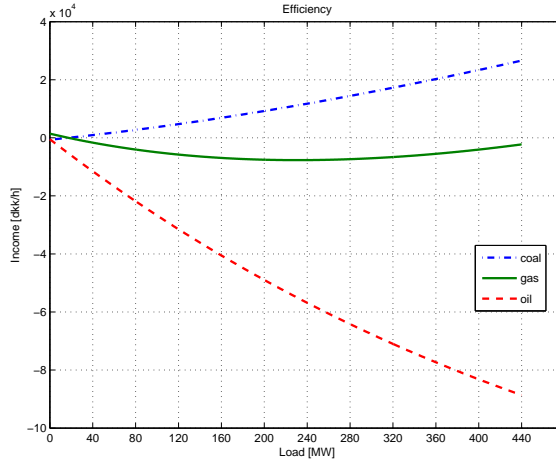


Figure 4.4: Graph of the income from the efficiency objective of the three different fuels. The horizontal axis illustrates the plant production in MW and the vertical axis denotes the income per hour, $\frac{dkk}{h}$.

3.2 Controllability Objective

A power plant is not only paid by the amount electricity produced but also the capability to change production as the available power always needs to fit the current demand of the electrical market. The ability to change production has, therefore, also a certain monetary value or income for a power plant. An expense associated to controllability is the fluctuations in the production, i.e., if a plant produces too little or too much power it is penalized.

The changes possible with the plant considered is depicted in Figure 4.5, i.e., when running the plant in the interval $[0MW, 200MW]$ and $[360MW, 400MW]$ it is possible to change the load with $2\frac{MW}{min}$ and in the interval $[200MW, 360MW]$ it is possible to change the load with $4\frac{MW}{min}$ and $8\frac{MW}{min}$ for coal and gas/oil respectively. These limits are set from the ability to control the different fuels and temperature constraints in the boiler, i.e., in order not to stress the metal in the boiler temperature gradients need to be under a certain limit which is ensured by using these limits. Functions describing the possible

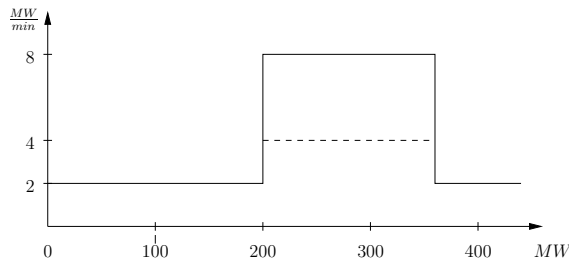


Figure 4.5: Possible load changes given certain running load (solid: gas/oil, dashed: coal.)

change for coal, $h_c(l)$, and oil and gas, $h_{go}(l)$, are defined as

$$h_c(l) = \begin{cases} \frac{0.033 \frac{MW}{s}}{l}, & 0 < l < 200 \\ \frac{0.067 \frac{MW}{s}}{l}, & 200 < l < 360 \\ \frac{0.033 \frac{MW}{s}}{l}, & 360 < l < 400 \end{cases} \quad (4.2)$$

$$h_{go}(l) = \begin{cases} \frac{0.033 \frac{MW}{s}}{l}, & 0 < l < 200 \\ \frac{0.133 \frac{MW}{s}}{l}, & 200 < l < 360 \\ \frac{0.033 \frac{MW}{s}}{l}, & 360 < l < 400, \end{cases} \quad (4.3)$$

where l is the load in MW .

The monetary value of the ability to change load has been determined from an internal DONG Energy document stating that it is possible to earn $1000000 \frac{dkk}{MW/min}$ each year from this ability. The expense associated to the noise in the output of the system is considered to be proportional to the variance in the output. Furthermore, the variance is assumed to be proportional to the load of the plant. When using oil or gas the plant can be controlled better than when using coal, therefore, the variance of the three fuels have been estimated to $0.015 \frac{W^2}{W}$, $0.002 \frac{W^2}{W}$, and $0.003 \frac{W^2}{W}$ for coal, gas, and oil respectively.

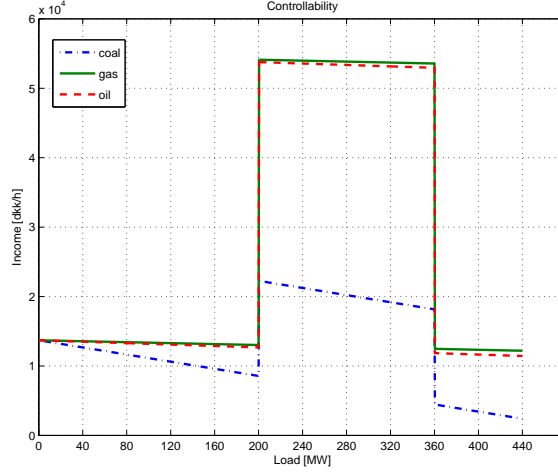


Figure 4.6: Graph of the income from the controllability objective of the three different fuels. The horizontal axis illustrates the plant production in MW and the vertical axis denotes the income per hour, $\frac{dkk}{h}$.

The conversion factor from variance to monetary value has been set to the same as for the income - at least in numerical sense. The income from controllability is calculated as

$$J_c(\mathbf{x}) = 6850 \frac{dkk}{s \cdot h} \begin{bmatrix} h_c(l) \mathbf{x}_{1:4} \\ h_{go}(l) \mathbf{x}_{5:20} \\ h_{go}(l) \mathbf{x}_{21:36} \end{bmatrix} - 6850 \frac{dkk}{MW \cdot h} \begin{bmatrix} \sigma_c^2 \mathbf{x}_{1:4} \\ \sigma_g^2 \mathbf{x}_{5:20} \\ \sigma_o^2 \mathbf{x}_{21:36} \end{bmatrix} \quad (4.4)$$

where \mathbf{x} is the load in MW of four coal mills, 16 gas burners, and 16 oil burners respectively and σ_c^2 , σ_g^2 , and σ_o^2 are the variances for coal, gas, and oil respectively as defined above. Figure 4.6 depicts graphs of the function J_c ; as seen gas yields the greatest income with regards to controllability - closely followed by oil.

3.3 Availability Objective

The last business objective considered in this example deals with availability which evaluates extra actuation power as it can be used to overcome possible faults in the system.

The available actuation power depends on how many actuators are used, the maximum possible actuation, and as mentioned the current actuation power. The maximum load possible with the different actuators is $532MW$, $452MW$, and $480MW$ for coal, gas, and oil respectively. Furthermore, when using coal four actuators is considered (the four coal mills) and for gas and oil 16 actuators are modelled (the individual burners), i.e., one actuator is sufficient for respectively $133MW$, $28.25MW$, and $30MW$ of production for coal, gas, and oil. Therefore, if a production of more than $133MW$, when using coal, is needed this implies that an additional actuator must be used. In this paper the available

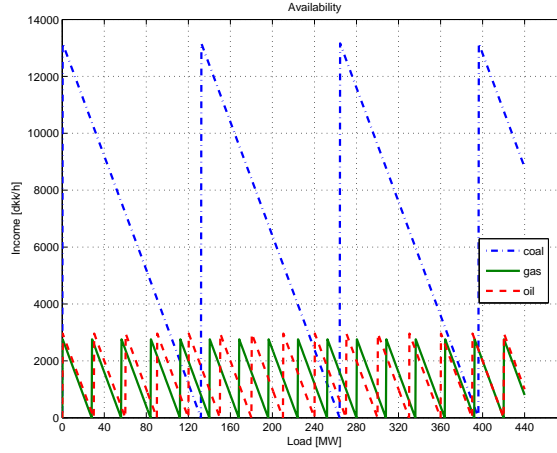


Figure 4.7: Graph of the income from the availability objective of the three different fuels. The horizontal axis illustrates the plant production in MW and the vertical axis denotes the income per hour, $\frac{dkk}{h}$.

actuation power is modelled as

$$h_a(\mathbf{x}) = \begin{bmatrix} 133 \frac{MW}{act} \cdot \mathbf{1}_{4 \times 1} - \mathbf{x}_{1:4} \\ 28.25 \frac{MW}{act} \cdot \mathbf{1}_{16 \times 1} - \mathbf{x}_{5:20} \\ 30 \frac{MW}{act} \cdot \mathbf{1}_{16 \times 1} - \mathbf{x}_{21:36} \end{bmatrix}, \quad (4.5)$$

where $\mathbf{1}_{a \times b}$ is a matrix with a rows and b columns all with ones, and \mathbf{x} is the load in MW of coal, gas, and oil respectively. The monetary value has been priced to $400 \frac{dkk}{MW \cdot h}$ which yields a maximum income of approximately half of what is possible from production.

The income from availability is calculated as

$$J_a(\mathbf{x}) = h_a(\mathbf{x}) \cdot 400 \frac{dkk}{MW \cdot h}. \quad (4.6)$$

Figure 4.7 depicts graphs of the function for availability for the three different fuels when the minimum number of actuators of are used.

3.4 Total Income

When choosing a fuel it is necessary to evaluate all of the objectives and as each of them returns a monetary value they can be added. The selection of which fuel to use can then be based on which fuel yields the greatest overall income. The total income is

$$J_t(\mathbf{x}) = J_e(\mathbf{x}) + J_c(\mathbf{x}) + J_a(\mathbf{x}), \mathbf{x} \in \mathbb{R}^{36}, \quad (4.7)$$

where $J_e(\mathbf{x})$, $J_c(\mathbf{x})$, and $J_a(\mathbf{x})$ are defined in (4.1), (4.4), and (4.6) respectively. Figure 4.8 shows the graph of the total income function, $J_t(\mathbf{x})$, when considering the three different fuels individually and when the minimum number of actuators are used. The function $J_t(\mathbf{x})$ for the production of the individual actuators in MW gives income per hour, $\frac{dkk}{h}$.

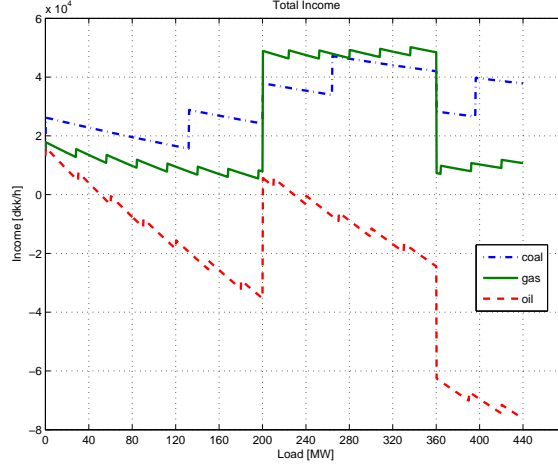


Figure 4.8: Graph of the total income of the three different fuels when combining the three objectives. The horizontal axis illustrates the plant production in MW and the vertical axis denotes the income per hour, $\frac{dkk}{h}$.

As seen in the figure coal yields the greatest income in low load and high load, however, there are loads where gas yields the greatest income and thus it is preferable. This is, however, evaluated by assuming that the minimum number of actuators is used e.g. when using coal if the total load is below $133MW$ only one actuator is used. This assumption is used to simplify the calculations of the total income.

3.5 Mixing Fuels

It is possible to investigate the monetary benefit of mixing fuels when considering the contribution in load from the three fuels (36 actuators) as a linear combination yielding the desired total load. The optimal cost of using a mixture of the 36 actuators can be calculated as

$$J_m(l) = \max_{\alpha \in \Delta} \quad \Sigma J_t(\alpha \times x) \quad (4.8)$$

$$s.t. \quad < \alpha, x > = l$$

where \times denotes the schur-product or element by element product,

$$\Delta = \left\{ \alpha \in \mathbb{R}^{36} \mid \sum_{i=1}^{36} \alpha_i = 1, \alpha_i \geq 0 \right\}, \quad (4.9)$$

x is the load in MW of coal, gas, and oil respectively, α denotes the mixing ratio of 4 coal burners, 16 oil burners and 16 gas burners, l is the desired total production load and $J_t(x)$ is defined in (4.7). By solving this optimization problem it is possible to choose which of actuators that should be used. If a actuator is not included in the optimal mix then it can be discarded.

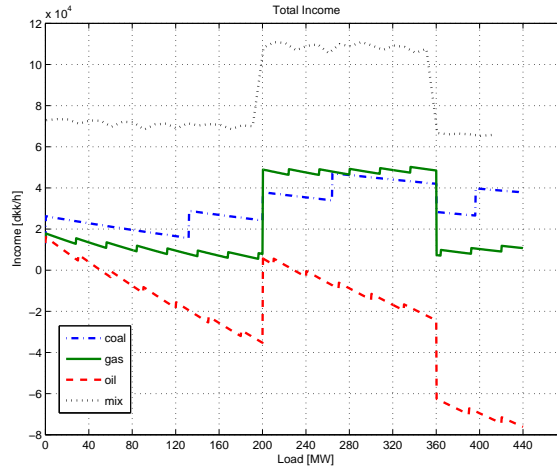


Figure 4.9: Graph of the total income possible when mixing oil and coal. The horizontal axis illustrates the plant production in MW and the vertical axis denotes the income per hour, $\frac{dkk}{h}$.

The optimization problem is formulated in the linear programming framework such that YALMIP³ can be used to solve the problem. The affine functions have been implemented by introducing auxiliary variables and equality constraints. Furthermore, an upper bound has been imposed on the income for extra available actuation power, i.e., J_a is bounded. The motivation is that given a certain market situation only a limited amount of extra actuation has a value.

A graph of $J_m(l)$ is depicted in Figure 4.9 along with the total income of the individual fuels. As seen in the figure it is possible to obtain a higher income when mixing the fuel types in an optimal manner. This is believed to be due to the extra controllability and availability obtain in the mixed fuel. The limit in availability was set to $150MW$.

The actuator configuration and loads of the individual actuators proposed by the algorithms at $100MW$, $200MW$, and $400MW$ is given in below.

100MW: At $100MW$ load production 5 actuators are used. 1 coal mill at $100MW$, 2 gas burners at $0MW$, and 2 oil burners at $0MW$.

200MW: At $200MW$ load production 10 actuators are used. 0 coal mills, 8 gas burners at $25MW$, and 2 oil burners at $0MW$.

400MW: At $400MW$ load production 11 actuators are used. 2 coal mills at $133MW$, 8 gas burners at $4 \cdot 28MW$, $23MW$, and $3 \cdot 0MW$, and 2 oil burners at $0MW$.

As seen the configuration changes as the load of the power plant is changed. Thus to find the actuators needed to run the power plant such that the greatest income is generated the configuration at all the desired loads must be evaluated and the minimum configuration can then be found. However, it would also be possible to evaluate if anything is

³YALMIP is a toolbox for Matlab which can be used defining and solving optimization problems

gained by e.g. adding 4 gas burners to a coal fired plant. In this example the optimal configuration is to equip the plant with 2 coal mills, 8 gas burners, and 2 oil burners.

4 Discussion and Future Work

This paper has presented a manually hierarchical breakdown of a boiler model, which is used to determine how business level objectives can be propagated to the individual subsystems. A business model of the top level objectives have been established using simple functions of the input and output of the system. Given a certain production load the functions return an income in $\frac{dkk}{h}$ which can be used to select which fuel to use under different operation conditions. Using the business model a maximization problem has been posed which yields the greatest possible income when mixing three different fuels. The maximization problem has been solved using the YALMIP toolbox to find the optimal actuator configuration at different production loads.

Future work include developing formal methods which can be used for propagating the business objectives and determining how different sets of sensors and actuators should be evaluated such that an optimal selection can be performed.

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5 Erratum

- The units $\frac{dkk}{MW_h}$ on page 43 is wrong and should be dropped completely (in fact all units should be dropped).
- Equation (4.1) should read as follows

$$\mathbf{J}_e(\mathbf{x}) = 200\mathbf{x} - \begin{bmatrix} (\mathbf{x}_{1:4} + 4 \cdot \mathbf{1}_{4 \times 1})72 \times (0.00018\mathbf{x}_{1:4} + 0.44 \cdot \mathbf{1}_{4 \times 1}) \\ (\mathbf{x}_{5:20} - 5 \cdot \mathbf{1}_{16 \times 1})104 \times (0.00031\mathbf{x}_{5:20} + 0.37 \cdot \mathbf{1}_{16 \times 1}) \\ (\mathbf{x}_{21:36} + \mathbf{1}_{16 \times 1})180 \times (0.00018\mathbf{x}_{21:36} + 0.37 \cdot \mathbf{1}_{16 \times 1}) \end{bmatrix}$$

where $\mathbf{J}_e \in \mathbb{R}^{36}$ is a vector of the efficiency objective of the individual actuators, \times is element-wise multiplication (schur product), $\mathbf{h}_{i:j}$ is a vector with elements i through j of \mathbf{h} , $\mathbf{x} \in \mathbb{R}^{36}$, and $\mathbf{1}_{i \times j}$ is a matrix with i rows and j columns and all elements are ones.

- Equation (4.4) should read as follows

$$\mathbf{J}_c(\mathbf{x}) = 6850 \begin{bmatrix} h_c(l)\mathbf{x}_{1:4} \\ h_{go}(l)\mathbf{x}_{5:20} \\ h_{go}(l)\mathbf{x}_{21:36} \end{bmatrix} - 6850 \begin{bmatrix} \sigma_c^2 \mathbf{x}_{1:4} \\ \sigma_g^2 \mathbf{x}_{5:20} \\ \sigma_o^2 \mathbf{x}_{21:36} \end{bmatrix}$$

where $\mathbf{J}_c \in \mathbb{R}^{36}$ is a vector of the controllability objective of the individual actuators,

$$l = \sum_{i=1}^{36} x_i,$$

and h, h_{go} and σ_k , $k = \{c, g, o\}$ are as given in the paper

- Equation (4.5) should read as follows

$$\mathbf{h}_a(\mathbf{x}) = \begin{bmatrix} 133 \cdot \mathbf{1}_{4 \times 1} - \mathbf{x}_{1:4} \\ 28.25 \cdot \mathbf{1}_{16 \times 1} - \mathbf{x}_{5:20} \\ 30 \cdot \mathbf{1}_{16 \times 1} - \mathbf{x}_{21:36} \end{bmatrix},$$

and as a consequence (4.6) should be

$$\mathbf{J}_a(\mathbf{x}) = \mathbf{h}_a(\mathbf{x}) \cdot 400,$$

where $\mathbf{J}_a \in \mathbb{R}^{36}$ is a vector of the availability objective of the individual actuators.

- The total income in (4.7) should then yield

$$\mathbf{J}_t(\mathbf{x}) = \mathbf{J}_e(\mathbf{x}) + \mathbf{J}_c(\mathbf{x}) + \mathbf{J}_a(\mathbf{x}), \mathbf{x} \in \mathbb{R}^{36},$$

where $\mathbf{J}_t \in \mathbb{R}^{36}$ is a vector of the total income of the individual actuators.

- As a consequence the mixing of fuels should be change, i.e., (4.8) and (4.9) should be changed to

$$J_m(l) = \max_{\alpha \in \Delta} \sum_{i=1}^{36} J_{t_i}(\alpha \times \mathbf{x}_m)$$
$$s.t. \quad \langle \alpha, \mathbf{x}_m \rangle = l$$

where \times denotes the schur-product or element by element product, $\mathbf{x}_m \in \mathbb{R}^{36}$ is a vector of the maximum of each actuator, and

$$\Delta = \left\{ \alpha \in \mathbb{R}^{36} \mid \sum_{i=1}^{36} \alpha_i = 1, \alpha_i \geq 0 \right\},$$

with α_i denoting the mixing ratio of each actuator, i.e., $\alpha_i x_{m_i}$ gives the production from actuator i .

Paper B

Optimal Usage of Coal, Gas, and Oil in a Power Plant

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The layout has been revised

Abstract

This paper addresses the problem of an optimal actuator selection when economic value is considered. The objective is to minimize the economical cost of operating a given plant. The problem has been formulated using mathematic notions from economics. Functionals describing the business objectives of operating a power plant has been established. The selection of actuator configuration has been limited to the fuel system which in the considered plant consists of three different fuels - coal, gas, and oil. The changes over 24 hours of operation is established and a strategy for using a plant utilizing the three fuels is developed which will yield a greater profit than a coal fired plant.

1 Introduction

The requirements for a complex process control system are usually derived from a top level (business) requirement to the entire system which is to maximize the income or profit of the company. However, the requirements specification for the process control system rarely includes profit maximization directly and instead the designer works with requirements to settling time, rise time, bandwidth, disturbance rejection etc., as these are easy to evaluate through simulation and well defined with respect to transfer functions and the pole placement of the closed loop system. All of these measures assume that a set of actuators and sensors is given. However, the choice of actuators and sensors influences the cost and performance of the system greatly - this will be addressed in this paper.

The selection of sensors and actuators has, to a great extent, depended on the designer's system knowledge and experience, however, in recent years more focus has been payed to developing tools to aid the designer during this phase as processes are becoming more complex and difficult to assess. One such tool is the Relative Gain Array (RGA), which can be used to pair inputs and outputs in a multiple input multiple output system to enable a decentralized control (single input single output control) [1, page 90]. Further advances using RGA have been examined in [2] where it is generalized to multiple output multiple input control structures.

The placement of sensors and actuators has been studied for different specific applications especially flexible structures in the aerospace-industry for which the methods are usually based on search algorithms, however these methods are difficult to generalize to other applications [3] as they consider the physical placement of actuator along a vibrating beam.

More general purpose methods for selecting and placing sensors and actuators have been evaluated in [4] and [5], which include e.g. methods relying on controllability measures such as state reachability and more sophisticated methods using robust performance measures. It is also concluded in [4] that the choice of sensors and actuators dictates the expenses for hardware, implementation, operation, and maintenance.

The methods mentioned above do not directly consider the cost/profit associated with the selection of actuators and sensors. The economical cost of sensors and actuators has, on the other hand, been considered in the selection method presented in [6], where the precision of a sensor or an actuator is assumed to be proportional to its cost. By introducing a bound on the economical cost of the instrumentation it is possible to formulate the design problem as a convex optimization. This helps the designer to select the right

instrumentation. However, this method only considers the implementation cost and not the operational cost which in many cases is the main concern for minimization [7].

As the requirements for a process control system usually are derived from business objectives it would be natural to include these business objectives when configuring the sensor/actuator layout of a plant. An attempt of this has been presented in [8] where functionals describing the business objectives are maximized. In [8] heuristics was used to solve the problem and the functionals encapsulated both the economical value and business objective measures.

The work in [9] was extended to utilize notions from production economics. When viewing a market from the production perspective one usually defines a number of companies and the goods they are capable of producing. The firms are viewed as a black box able to transform inputs to outputs [10]. In [9] this approach was used by formulating functionals which describe DONG Energy's¹ objectives for a power plant, which is a complex process control system, as outputs and the amount of fuel used as input. The price of producing the output and price of using the fuel/input was described by approximating data from a power trading market.

This paper will use two of the three business functionals from [9] which the third objective, availability, is discarded as it does not depend on the actuator selection. The results are in this paper, furthermore, extended to real price and demand data and a scenario with only partial production capabilities in the coal and gas system will be considered, which is interesting as most coal plants are started using gas or oil. This paper shows that a power plant capable of using coal as well as gas and oil will be able to generate a larger profit during normal operating conditions than a purely coal fired plant - in particular June 29th, 2008 is considered, however, the result would be similar for any given day. During this day a profit increase of 12% is possible.

The work in this paper should be seen in relation to the Plug and Play Process Control (P3C) project [11]. The P3C project is investigating how to develop control algorithms and infrastructure to make plug-and-play, as known from the personal computer industry, possible in process control system. However, when should new hardware be plugged in and what are the benefits? These kinds of questions are investigated in this paper using a power plant as an example, i.e., two questions are addressed; when should "new" hardware (fuel systems) be used and what is the benefit (economical profit).

1.1 Outline

The plant considered in this work is presented in Section 2 and then the problem is formulated. Two of DONG Energy's business objectives - Efficiency and Controllability - are described in Section 3 as static models for three different actuator systems; coal, gas, and oil. In Section 4 the problem of profit maximization is solved using the static models and the results are presented. The static models are expanded in Section 5 to include the dynamic nature of electricity prices and production reference during 24 hours. The dynamic formulation is solved in Section 6 and it is shown that a power plant with multiple fuels can provide a greater profit than a traditional coal fired power plan. Finally a discussion about the results is brought in Section 9.

¹DONG Energy is a Danish energy supplier

2 Problem Formulation

The problem in this work has been formulated in collaboration with DONG Energy - a Danish power company. The goal of any company is to maximize its profit and for DONG Energy the profit maximization has been divided into four individual business objectives which can be described by Efficiency, Controllability, Availability, and Life Time (to simplify the model only the first two objectives are considered in this work) which will be defined in Section 3. The problem formulated is based on a model of a coal fired boiler - a vital component of a power plant - which is augmented with two additional fuels system; gas and oil.

2.1 Plant Description

The power plant considered in this paper consists of the following components:

Fuel system The fuel system prepares the different fuels for burning, e.g. the coal mills grind the coal to small dust particles which burn quickly and efficiently.

Burners The burners deliver the fuel to specific places in the boiler such that the heat transfer is maximized.

Boiler The boiler is a module where the fuels are burned thereby heat is delivered to the evaporator.

Evaporator The evaporator is fed with water, which is evaporated under high pressure by the heat from the burners.

Superheater The superheater (super) heats the steam from the evaporator.

Economizer The economizer uses some of the remaining heat in the flue gas to preheat the feed water before it enters the evaporator.

The individual parts of the model are illustrated in Figure 5.1.

The power plant has the possibility to use three different fuels which have certain advantages and disadvantages e.g. gas is easy to control but an expensive fuel. Some of the characteristics of the different fuels are:

Coal is advantageous when considering the price per stored energy, however, it is difficult to control as unmeasurable fluctuations in the coal flow are introduced by the coal mill when the coal is ground to coal dust. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind and dry the coal which needs to be considered.

Gas is more expensive than coal and energy is not converted to steam as efficient with gas as with coal due to the layout of the chosen boiler. However, gas arrives at the power plant under high pressure which is lowered using a turbine generating electrical energy. Furthermore, gas is much easier to control as it is possible to measure the flow.

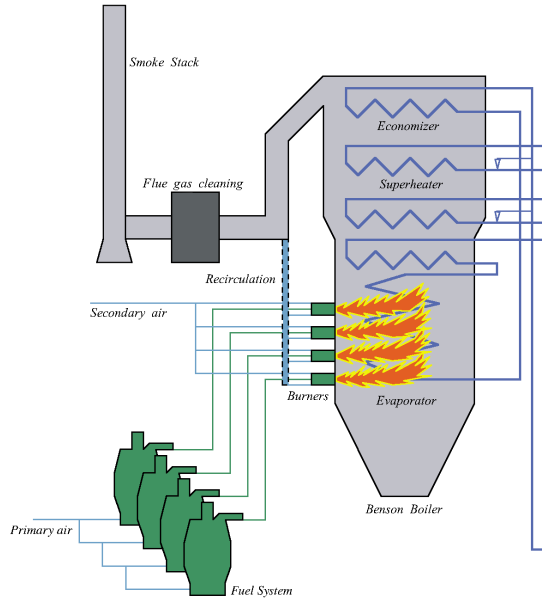


Figure 5.1: Power plant model including the different modules from fuel processing to steam delivery.

Oil is, with the current market prices, the most expensive of the three fuels and has to be heated before entering the boiler. This process demands energy itself. Nevertheless, oil is considered in this work as it is possible to measure the oil flow into the boiler making it easy to control. Furthermore, oil is present in most existing coal fired plants as oil is used in the period of starting the plant.

2.2 Problem

The focus of this work is to derive a mixture of the three fuels, described above, which will yield the greatest profit under consideration of the two business objectives; Efficiency and Controllability. The idea is to develop simple models of the business objectives to evaluate if there is an economical gain of mixing fuels. If it is advantageous to mix fuels a strategy for using the fuels will be developed. The idea in this work is not to develop controller for the plant as it is assumed this is done or will be done by other known methods.

3 Static Plant Model

In the sequel, models of the efficiency and controllability objectives will be derived for the input of coal, gas, and oil. Furthermore, the input and output spaces are described. The input space is a polytope (more precisely a simplex) in a Euclidean space. Its coordinates are flows of coal, gas, and oil. The power plant production is characterized by a map taking the fuel flow into a pair of production objectives: efficiency - actual power

production in $[MW]$ and controllability - ability to adjust the production to instantaneous needs of the market. The production objectives have associated price which is related to markets demands. The profit can now be calculated as the revenue from efficiency and controllability minus the expenses of using fuel. The article applies static optimization to devise a fuel utilization plan for coal, gas and oil such that the profit is maximal and the demand for production is satisfied.

Let \mathbb{R}_+^3 denote the positive quadrant in \mathbb{R}^3 , i.e., $\mathbb{R}_+^3 = \{\mathbf{v} \in \mathbb{R}^3 | \mathbf{v} \geq \mathbf{0}\}$ where the inequality is to be understood coordinate wise (this notation will be used throughout this work).

The input space X is now given by

$$X = \{\mathbf{v} \in \mathbb{R}_+^3 | 0 \leq (\mathbf{v} | \mathbf{u}) \leq c\}, \quad (5.1)$$

where $(\cdot | \cdot)$ is the Euclidean inner product, and the vector $\mathbf{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ with $\mathbf{u} > \mathbf{0}$ and scalar $c \in \mathbb{R}$ are to be determined later. Note that X is the 3-simplex (in \mathbb{R}_+^3) with vertices $\mathbf{0}$, $(c/u_1, 0, 0)$, $(0, c/u_2, 0)$, and $(0, 0, c/u_3)$. Each input

$$\mathbf{x} = (x_c, x_g, x_o) \in X, \quad ([kg/s], [kg/s], [kg/s]),$$

to the system describes the flow of coal, gas, and oil respectively, measured in kilogram per second $[kg/s]$ (brackets, $[\cdot]$, will be used for denoting units throughout this work). In the sequel we let $\mathcal{I} = \{c, g, o\}$ where the elements of the index set \mathcal{I} refers to the three different fuels. Occasionally the identification $(c, g, o) = (1, 2, 3)$ will be used.

The output space $Y = Y_1 \times Y_2$ is a subset of \mathbb{R}^2 where each output²

$$\mathbf{y} = (y_e, y_c) \in Y, \quad ([MW], [MW/s]),$$

of the system describes one of the two objectives; efficiency and controllability, respectively, i.e., y_e is a measure of the efficiency and y_c is a measure of the controllability. Both of these quantities contain contributions from coal, gas, and oil as will be explained next, where simple functions describing these two business objectives at steady state are derived.

3.1 Efficiency

The efficiency objective, y_e , expresses how much electricity is produced from a certain amount of fuel. Three affine functions describing the contribution of the individual fuels to the efficiency objective have been established using measurement data from two Danish power plants. These function are given by

$$\begin{aligned} y_{ec}(x_c) &= e_c x_c + e'_c, \\ y_{eg}(x_g) &= e_g x_g + e'_g, \\ y_{eo}(x_o) &= e_o x_o + e'_o, \end{aligned}$$

where

$$(e_c, e_g, e_o) = (10.77, 18.87, 15.77),$$

²MW is an abbreviation for Mega Watt.

are measures of how much energy is stored in the individual fuels (in³ $[MJ/kg]$) and

$$(e'_c, e'_g, e'_o) = (-1.76, 1.85, -0.37),$$

are the own-consumptions of the different fuels (in $[MW]$) as explained in Section 2.1. The values above have been established using measurement data provided by DONG Energy.

The total amount of efficiency (at steady state) is described by the function

$$X \rightarrow Y_1; \mathbf{x} \mapsto y_e(\mathbf{x}) = \sum_{i \in \mathcal{I}} y_{ei}(x_i) = (\mathbf{x}|\mathbf{u}) + c', \quad (5.2)$$

where $c' = \sum e'_i$ and $\mathbf{u} = (e_c, e_g, e_o)$ which also should be used in (5.1). The constant c in (5.1) can now be determined by $c = 400 - c'$, where 400 refers to the maximum efficiency (in $[MW]$) produced by the plant and c' is an expression of the own-consumption of the complete plant which is lost in the electricity production. Finally Y_1 can be determined by $Y_1 = (0, 400]$.

3.2 Controllability

The controllability objective, y_c , gives a measure of how fast the production of electricity can be changed. Allowed change in the production is limited to a certain gradient depending on the current efficiency, y_e . The reason for this limit is a compliance to maximum temperature gradients in the boiler (the temperature gradients have not been explicit modelled and are therefore indirectly considered this way). When running the plant in ranges $0 [MW]$ to $200 [MW]$ and $360 [MW]$ to $400 [MW]$ it is allowed to change production by $0.133 [MW/s]$ independent of fuel. However, in the range $200 [MW]$ to $360 [MW]$ the allowed changes are dependent of which fuel is used. If coal is used it is allowed to change production by $0.267 [MW/s]$ and when using oil and gas the allowed change is $0.534 [MW/s]$. The changes allowed is modelled as piece-wise constant functions

$$h_i : Y_1 \rightarrow \mathbb{R} \quad ([MW] \mapsto [MW/s]), \quad i \in \{1, 2, 3\},$$

given by

$$h_i(y_1) = \begin{cases} 0.133 & y_1 \in (0, 200) \cup (360, 400] \\ 0.267 \cdot i & y_1 \in [200, 360] \end{cases} \quad i = 1, 2 \quad (5.3)$$

$$h_2 = h_3. \quad (5.4)$$

If a mixture of the three fuels are used it is assumed that the allowed change is a certain convex combination of the allowed change of the individual fuels. More precisely, the total amount of controllability is expressed by the function

$$X \rightarrow Y_2; y_c(x) = \sum_{i \in \mathcal{I}} y_{ci}(x), \quad (5.5)$$

³MJ is an abbreviation for Mega Joule.

where

$$\begin{aligned} y_{cc}(x) &= \frac{y_{ec}(x_c)}{y_e(x)} h_1(y_e(x)), \\ y_{cg}(x) &= \frac{y_{eg}(x_g)}{y_e(x)} h_2(y_e(x)), \\ y_{co}(x) &= \frac{y_{eo}(x_o)}{y_e(x)} h_3(y_e(x)). \end{aligned}$$

The values in this model have been established in collaboration with DONG Energy.

3.3 Prices

At steady state the cost of using input \mathbf{x} , revenue from production of output \mathbf{y} , and the profit of operating the power plant can now be determined. The above constructions yield a product (or output) function, \mathbf{y}_P , of the system given by

$$\mathbf{y}_P : X \rightarrow Y; \mathbf{x} \mapsto (y_e(\mathbf{x}), y_c(\mathbf{x})).$$

For the system, the growth of cost and growth of revenue are defined by the following functions⁴

$$\begin{aligned} g_C : X \rightarrow \mathbb{R}; \mathbf{x} &\mapsto (\mathbf{x}|\mathbf{p}_C) \text{ [DKK/s]}, \\ g_R : Y \rightarrow \mathbb{R}; \mathbf{y} &\mapsto (\mathbf{y}|\mathbf{p}_R) \text{ [DKK/s]}, \end{aligned}$$

with price vectors

$$\begin{aligned} \mathbf{p}_C &= (p_{C1}, p_{C2}, p_{C3}) = (1.20, 3.74, 6.00), \\ \mathbf{p}_R &= (p_{R1}, p_{R2}) = (0.16, 247), \end{aligned}$$

fixed and in units [DKK/kg] for p_{Ci} , [DKK/MWs] for p_{R1} , and [DKK/MW] for p_{R2} . The prices correspond to the maximum market prices June 29th, 2008 (see Section 5).

The growth of profit is defined by the function

$$X \times Y \rightarrow \mathbb{R}; (\mathbf{x}, \mathbf{y}) \mapsto g_R(\mathbf{y}) - g_C(\mathbf{x}),$$

which for the system yields

$$g_P : X \rightarrow \mathbb{R}; \mathbf{x} \mapsto g_R(\mathbf{y}_P(\mathbf{x})) - g_C(\mathbf{x}).$$

Hence the profit is given by

$$P : \mathbb{R}_+ \rightarrow \mathbb{R}; t \mapsto \int_0^t g_P(\mathbf{x}) d\tau.$$

⁴DKK is an abbreviation for the Danish currency.

4 Static Optimization

In the following we wish to find the optimal static fuel configuration, \mathbf{x}^* , such that the growth of profit, and thus the profit, is maximized. For a given efficiency $y_r \in Y_1$ we consider the maximum growth of profit

$$\max_{\mathbf{x} \in y_e^{-1}(y_r)} g_P(\mathbf{x}), \quad (5.6)$$

where we note that $y_e^{-1}(y_r)$ is the 2-simplex (in $X \subset \mathbb{R}_+^3$) with vertices

$$\begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0), \\ \mathbf{v}_2^* &= (0, (y_r - c')/u_2, 0), \\ \mathbf{v}_3^* &= (0, 0, (y_r - c')/u_3). \end{aligned} \quad (5.7)$$

Since g_P restricted to the set $\{\mathbf{x} \in X | \mathbf{x} \in y_e^{-1}(y_r)\}$ is affine, the optimal configuration is given by

$$\mathbf{x}^* = \arg \max_{\mathbf{x} \in y_e^{-1}(y_r)} g_P(\mathbf{x}) \in \{\mathbf{v}_i^*\}, \quad (5.8)$$

for each y_r , i.e.,

$$\max_{\mathbf{x} \in y_e^{-1}(y_r)} g_P(\mathbf{x}) \in \{g_P(\mathbf{v}_i^*)\},$$

and that we may describe the maximum growth of profit and the optimal configuration as functions of the efficiency by

$$Y_1 \rightarrow \mathbb{R}; y_r \mapsto \max_{\mathbf{x} \in y_e^{-1}(y_r)} g_P(\mathbf{x}), \quad (5.9)$$

$$Y_1 \rightarrow X; y_r \mapsto \arg \max_{\mathbf{x} \in y_e^{-1}(y_r)} g_P(\mathbf{x}). \quad (5.10)$$

Figure 5.2-top depicts the graph of (5.9), i.e., the maximum growth of profit versus the efficiency. The bottom figure depicts the graph of (5.10), i.e., the optimal configuration versus the efficiency where the values on the 2nd axis should be read with the identification $(1, 2, 3) = (\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*)$. As seen in the figure the optimal configuration is changed from using only coal to using only gas when the efficiency is in the range $[200, 360]$. The gradient of the growth of profit is negative when using gas which is caused by the higher gas price. However, the growth of profit caused by the controllability, y_c , still makes gas advantageous.

The results above suggests that gas should be used whenever the efficiency is in the range $[200, 360]$ and coal otherwise. However, things are not as obvious as it seems because the prices of the objective, p_R change during the day. These changes of the prices will be considered in the following section.

5 Dynamic Plant Model

The electricity production of a power plant is not constant during the year or even during 24 hours. However, prediction of the demand of power 24 hours into the future makes it possible to plan production ahead of time. During this planning for the entire electrical

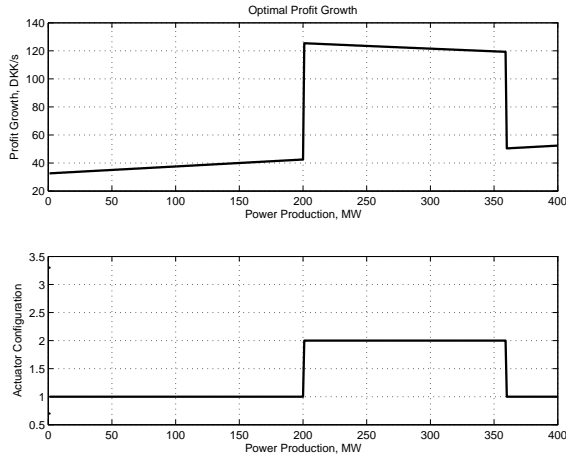


Figure 5.2: Top: Optimal profit growth. Bottom: Fuel configuration (2nd axis should be read with the identification $(1, 2, 3) = (v_1^*, v_2^*, v_3^*)$).

grid (consisting of multiple power plants throughout Denmark) a production plan is fitted to the capabilities of the individual plants, i.e. a production plan (y_e reference) is delivered to each power plant. The prices of efficiency and controllability are also established during this planning. In the following these changes will be described and models of the effects will be derived.

5.1 Production Plan

The total power production in West Denmark over 24 hours during 30 days is depicted in Figure 5.3. The data used to generate this plot has been obtained from Nord Pool⁵ and the graphs for the individual days have been normalized by the maximum production during that day. In West Denmark there are multiple power plants and the total power production is obviously a sum of the production of these individual plants. It is expected that the production plan for the individual plants follows the trends in Figure 5.3. Hence, the production is low at night and in the morning around 6:00 there is a large increase in production and finally, in the afternoon the production fluctuates a bit. In this work we will consider a particular day, where the relevant data has been provided by DONG Energy and Nord Pool. However, the methods presented can be used for any given day of the year. The production plan for the day considered in this work is depicted in Figure 5.4. The graph depicts the production from midnight June 29th, 2008 and 24 hour ahead. As seen in the figure the production is rather low during the night but at 6:00-7:00 in the morning there is a steep gradient caused by the increase in consumption when people and companies start to use electricity. During the afternoon and evening some fluctuations are seen. The production plan is modelled as an approximation of the graph depicted in Figure 5.4 and is denoted

$$t \mapsto y_r(t). \quad (5.11)$$

⁵Nord Pool is a marketplace for trading power contracts (www.nordpool.dk).

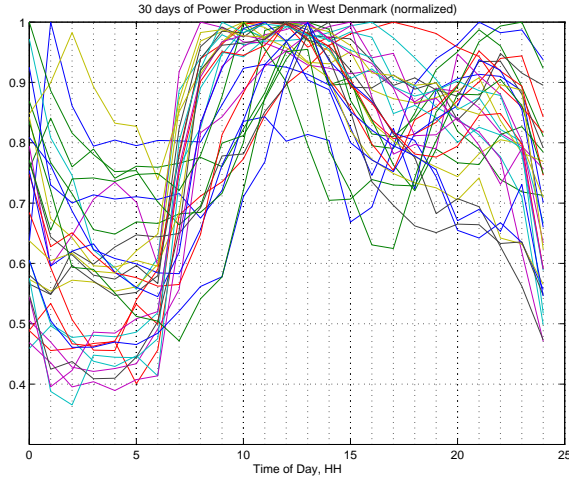


Figure 5.3: Total power production over 24 hours during 30 days. The data used to generate this plot has been found on www.nordpool.dk.

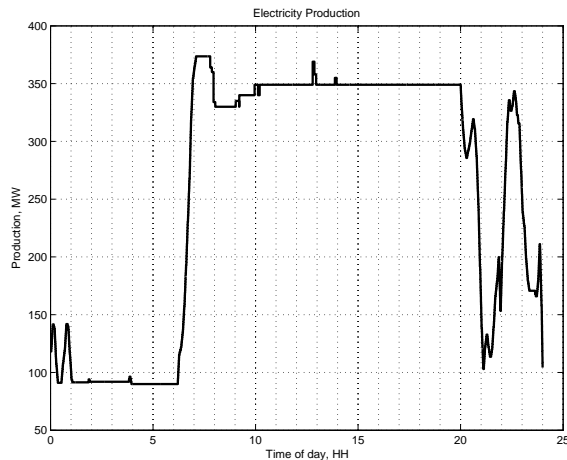


Figure 5.4: A production plan over 24 hours June 29th, 2008. The data used to generate this plot has been provided by DONG Energy.

5.2 Efficiency Price

The price of electricity, p_{R1} , changes during the day as the demand changes, i.e., during the middle of the day when the demand is greatest the price is also higher than during the early morning. The trading prices for electricity over 24 hours during 30 days is depicted in Figure 5.5 where the average is depicted as well.

The electricity price from the day considered in this work (June 29th, 2008) is depicted in Figure 5.6 where the data has been found at the archive at Nord Pool. The price is

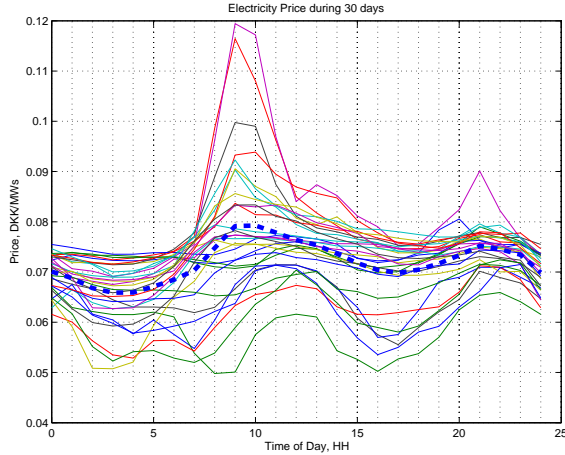


Figure 5.5: The efficiency price over 24 hours during 30 days and average price (thick dashed). The data used to generate this plot has been found on www.nordpool.dk

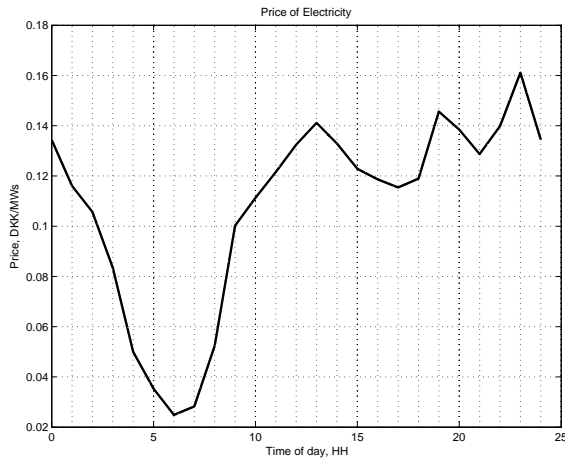


Figure 5.6: The efficiency price during the 29th of June 2008. The data used to generate this plot has been found on www.nordpool.dk

modelled as an approximation of this graph and is denoted

$$t \mapsto p_{R1}(t). \quad (5.12)$$

5.3 Controllability Price

Large gradients in the production plan, as seen in Figure 5.4 around 6:00-7:00, yield a high price on controllability as it is likely that some plants are not capable of generating the gradients needed.

According to DONG Energy, the controllability price would, in general, be related to the

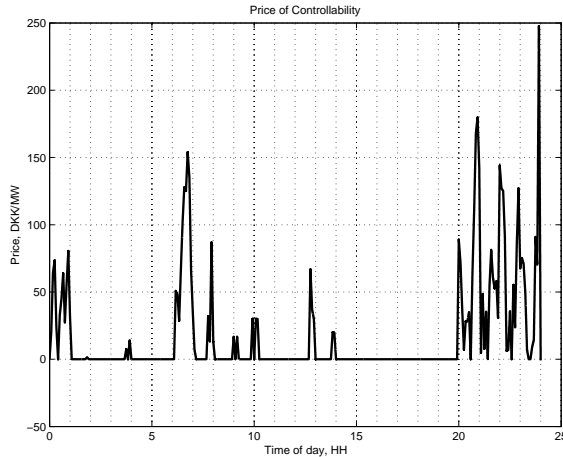


Figure 5.7: Modelled controllability price during June 29th, 2008. The data in this plot has been established in collaboration with DONG Energy.

derivative of the production plan. Hence, the price is higher during the periods in the morning and afternoon/evening where there exists steep gradients as seen in Figure 5.4. The approximation of the controllability price is defined as

$$t \mapsto p_{R2}(t) = \beta \left| \frac{d}{dt} y_e(t) \right|, \quad (5.13)$$

where $\beta = 1000$ is a factor which has been determined in collaboration with DONG Energy. We remark that established model is simplifying a complicated price model but is considered sufficient for this work. The modelled controllability price, p_{R2} , is depicted in Figure 5.7.

5.4 Fuel Price

Obviously the fuel prices change over time, however, these changes are slow compared to the changes described in the previous sections. The time span is a matter of weeks and is therefore, compared to the above, roughly constant and therefore the fuel prices given in Section 3 are used.

5.5 Discussion of Prices

The average price for efficiency is $0.11 [DKK/MW s]$ and the average price for controllability is $17.2 [DKK/MW]$ which might seem as a large difference or an unrealistic high price on controllability. However, the values of the efficiency measure and controllability measure are also different as the efficiency output is in the range $(0, 400]$ and controllability output is in the range $[0.133, 0.534]$. At a load of $300 [MW]$ the instantaneous income⁶ from efficiency is $32 [DKK/s]$ and from controllability the instantaneous

⁶Here the term instantaneous income is used instead of growth of profit as only the revenue of efficiency and controllability is considered.

income is between 4.6 and 9.2 $[DKK/s]$ (using the average prices). At 6:30 the instantaneous incomes are 3.9 $[DKK/s]$ and 11 $[DKK/s]$ for efficiency and controllability, respectively. On average, that is, the determining factor for revenue is the efficiency measure but at certain periods during the day the controllability measure becomes significant.

6 Fuel Selection in Dynamic Case

In the following the static optimization problem given in Section 4 is expanded to include the time dependence described in Section 5. The growth of profit and the profit is maximized during 24 hours of operation.

Since the prices on the outputs are time dependent the growth of revenue for the system will now be defined by

$$g_R : Y \times \mathbb{R}_+ \rightarrow \mathbb{R}; (\mathbf{y}, t) \mapsto (\mathbf{y} | \mathbf{p}_R(t)),$$

where $\mathbf{p}_R(t) = (p_{R1}(t), p_{R2}(t))$ with the coordinate functions as defined in (5.12) and (5.13).

Hence, the growth of profit will be time dependent and given by

$$X \times Y \times \mathbb{R}_+ \rightarrow \mathbb{R}; (\mathbf{x}, \mathbf{y}, t) \mapsto g_R(\mathbf{y}, t) - g_C(\mathbf{x}),$$

which for the system yields

$$g_P : X \times \mathbb{R}_+ \rightarrow \mathbb{R}; (\mathbf{x}, t) \mapsto g_R(\mathbf{y}_P(\mathbf{x}), t) - g_C(\mathbf{x}). \quad (5.14)$$

The objective is now to let the efficiency, y_e follow some predefined time dependent reference signal (see Section 5.1), i.e., $y_e = y_r(t)$.

For given t^* we consider the maximum growth of profit

$$\max_{\mathbf{x} \in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*).$$

Hence, as in Section 4 we obtain

$$\mathbf{x}^*(t^*) = \arg \max_{\mathbf{x} \in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*) \in \{\mathbf{v}_i^*(t^*)\},$$

and for each t^*

$$\max_{\mathbf{x} \in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*) \in \{g_P(\mathbf{v}_i^*(t^*), t^*)\},$$

where the \mathbf{v}_i^* s are as in (5.7) with y_r replaced by $y_r(t^*)$. The optimal fuel configuration is now described by the curve

$$\mathbb{R}_+ \rightarrow \{\mathbf{v}_i^*\}; t \mapsto \mathbf{x}^*(t), \quad (5.15)$$

so the maximum growth of profit and maximum profit as functions of time are given by

$$G_P : \mathbb{R}_+ \rightarrow \mathbb{R}; t \mapsto g_P(\mathbf{x}^*(t), t), \quad (5.16)$$

$$P : \mathbb{R}_+ \rightarrow \mathbb{R}; t \mapsto \int_0^t G_P(\tau) d\tau. \quad (5.17)$$

In the following results the real data sets have been used for $y_r(t)$, $p_{R1}(t)$, and $p_{R2}(t)$. Figure 5.8 top shows the graph of G_P , i.e., the maximum growth of profit versus time and the bottom figure depicts the graph of (5.15), i.e., the optimal fuel configuration versus time, where the identification $(1, 2, 3) = (v_1^*, v_2^*, v_3^*)$ is used.

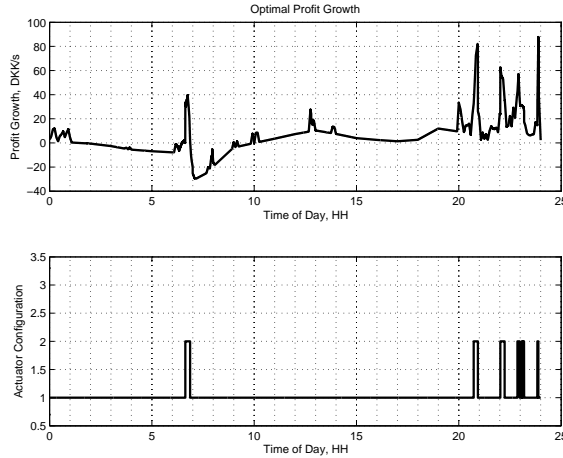


Figure 5.8: Top: Growth of profit. Bottom: Optimal fuel configuration. Both plotted over 24 hours of operation June 29th, 2008.

The growth of profit is, as seen in the figure, negative during the early morning hours where the price of efficiency is low (see Figure 5.6). Furthermore, some spikes are present around 6:00-7:00 and between 20:00-24:00 which are caused by shifting fuel from coal to gas and vice versa. As depicted in the figure, coal is used during most of the day. The use of coal at night is partially expected from the static optimization as the efficiency reference is low, however, due to a low price on controllability during the middle of the day coal is used instead for gas as expected from the static optimization. In the evening gas is used to cope with the changes in the demand of electric power.

In Figure 5.9 the graph of P , defined in (5.17), is depicted, i.e., the maximum profit versus time. The profit is low during the morning and actually negative most of the day until around 19:00, however, during the evening when the efficiency price is high the profit grows.

In Figure 5.10 the profit is compared to a plant using only coal. Plants using only gas or oil will at the end of the day have a deficit of respectively 1.4 and 5.5 million DKK and these are, therefore, not depicted. As seen in the figure the profit from the two plants are equal until around 7:00 where gas is used in the mixed fuel plant. The difference in profit is during the day enlarged and at the end of the day the gain by using a mixed fuel is around 40000DKK or 12% more compared to the plant using only coal.

7 Change of Parameters

In this section a discussion is made about how the results change when two of the parameters in the model of the plant are changed. The parameters considered are the controlla-

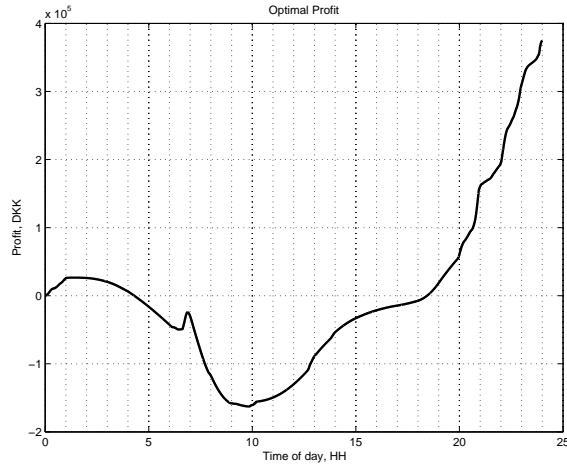


Figure 5.9: Optimal profit over 24 hours June 29th, 2008.

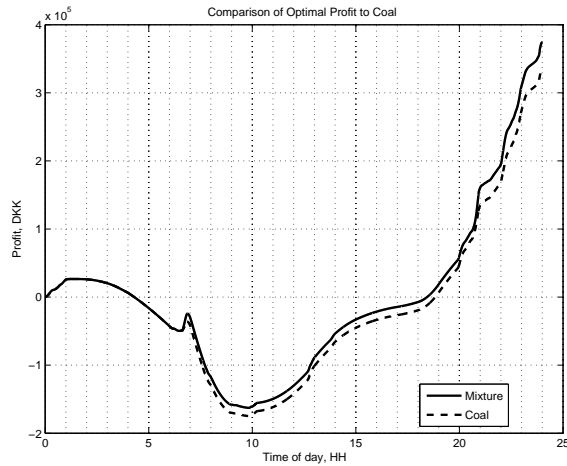


Figure 5.10: Profit for a plant using a mixture of fuels is compared to a plant using only coal over 24 hours of operation June 29th, 2008.

bility price and the production capabilities of oil and gas.

7.1 Controllability Price

This section discuss how the results are influenced by changing β in the controllability price (see (5.13)). If the fuel configuration in Figure 5.8 is compared to the controllability price in Figure 5.7 it can be observed that gas is chosen when the controllability price is above $100[\text{DKK}/\text{MW}]$ and thus changing β will influence how often and how long time gas is used. If β is enlarged it is expected that gas will be used more often and thus it will be more valuable to be able to use both gas and coal. The optimal actuator configuration

is depicted in Figure 5.11 where $\beta = 10000$ and $\beta = 100$ are used. As seen gas is not selected when $\beta = 100$ is used but as expected gas is selected more during the day when $\beta = 10000$.

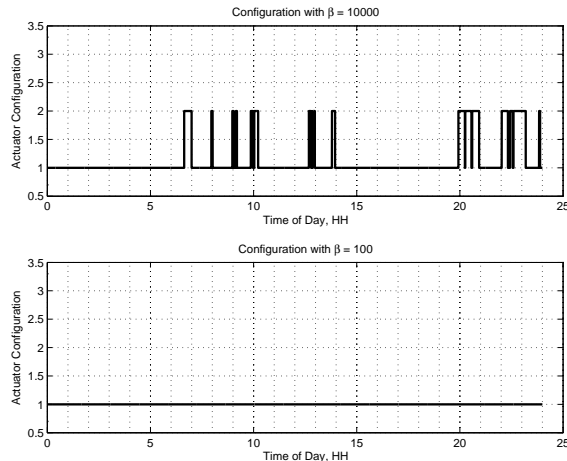


Figure 5.11: Optimal actuator configuration with $\beta = 10000$ and $\beta = 100$ over 24 hours of operation during the June 29th, 2008.

7.2 Partial Production Capabilities

The three different fuel systems considered in this work are comprised of multiple actuators, e.g. the coal system consists of four coal mills and the gas and oil system consists of 16 burners each. Furthermore, it can be argued that three systems capable of delivering fuel to full production might not be feasible as the cost of implementing this is large when 2/3 of the actuation power is not in use. Therefore, in this section it will be investigated how the result changes when the gas and oil systems only consist of 4 burners each, i.e., 25% of what is considered in Section 6. This configuration is interesting because the burners are usually implemented in sets of four and at least one set is present in existing coal fired plants as it is necessary in order to start up the plant.

The solution to this problem follows the procedure from the previous sections where $y_e^{-1}(y_r)$ in (5.7) changes from a simplex to a polytope of dimension 2 depending on the

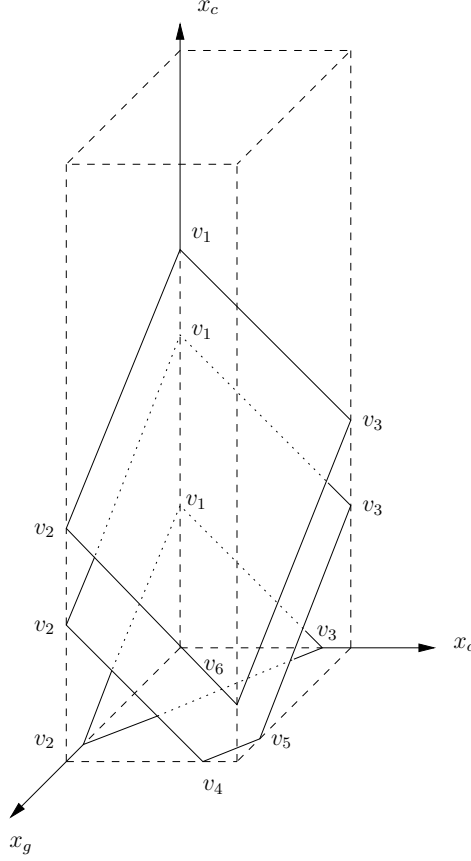


Figure 5.12: Illustration of the input space where the optimal configuration is located on one of the vertices.

value of y_r . More precisely the vertices of $y_e^{-1}(y_r)$ becomes

$$\begin{aligned}
 & \left. \begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= (0, (y_r - c')/u_2, 0) \\ \mathbf{v}_3^* &= (0, 0, (y_r - c')/u_3) \end{aligned} \right\} y_r \in (0, 100] \\
 & \left. \begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= ((y_r - 100 - c')/u_1, (100 - c')/u_2, 0) \\ \mathbf{v}_3^* &= ((y_r - 100 - c')/u_1, 0, (100 - c')/u_3) \\ \mathbf{v}_4^* &= (0, (100 - c')/u_2, (y_r - 100 - c')/u_3) \\ \mathbf{v}_5^* &= (0, (y_r - 100 - c')/u_2, (100 - c')/u_3) \end{aligned} \right\} y_r \in (100, 200] \\
 & \left. \begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= ((y_r - 100 - c')/u_1, (100 - c')/u_2, 0) \\ \mathbf{v}_3^* &= ((y_r - 100 - c')/u_1, 0, (100 - c')/u_3) \\ \mathbf{v}_6^* &= ((y_r - 200 - c')/u_1, (100 - c')/u_2, (100 - c')/u_3) \end{aligned} \right\} y_r \in (200, 400]
 \end{aligned}$$

The vertices and thus the potential optimal configurations are illustrated in Figure 5.12; it arises as the intersection between the efficiency plane and the constraint set.

The results from the static optimization are depicted in Figure 5.13 where the top graph is the growth of profit as a function of the efficiency. The bottom graph depicts the fuel configuration with the identification $(1, 2, 3, 4, 5, 6) = (v_1^*, v_2^*, v_3^*, v_4^*, v_5^*, v_6^*)$, with v_i^* defined as above. As the figure shows the oil system is now used in the interval

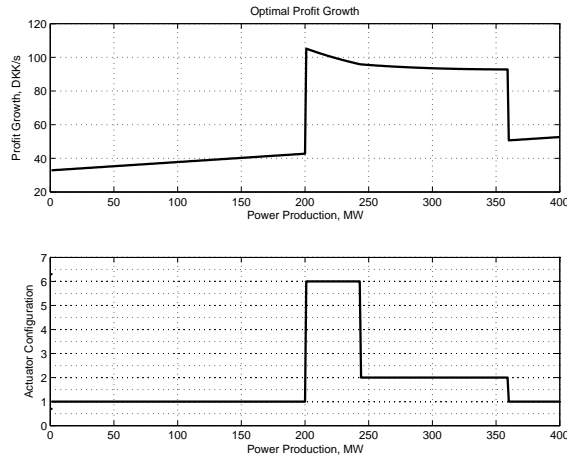


Figure 5.13: Top: Optimal profit growth with 25% production capabilities of gas and oil. Bottom: Fuel configuration (2nd axis should be read with the identification $(1, 2, 3, 4, 5, 6) = (v_1^*, v_2^*, v_3^*, v_4^*, v_5^*, v_6^*)$).

[200, 240].

The results of introducing the limit in the gas and oil system in the dynamic case are depicted in Figure 5.14, where the top graph is the profit during 24 hours of operation and the bottom graph is the fuel configuration with the identification as above. This is very similar to the results without the limit and it can be concluded that oil is not used at all. A limit of 25% of full production in gas and oil results in a gain of 16000DKK or 5% compared to the case of only using coal, i.e, a reduction of 75% in production capabilities of the two fuels results in a reduction of 60% of the net income.

8 Including Plant Dynamics

In this section a brief discussion will be made of the optimization problem when plant dynamics is considered.

First, let

$$Z = \{z = (z_1, z_2, \dots, z_9) \in \mathbb{R}^9 | (z_1, z_4, z_7) \in X\},$$

be an auxiliary state space, which is used when describing the dynamics of the fuel flows. The fuel flow, $x(t)$, into the power plant is governed by third order differential equations (these equations also include a simple model for the power plant dynamics). The control

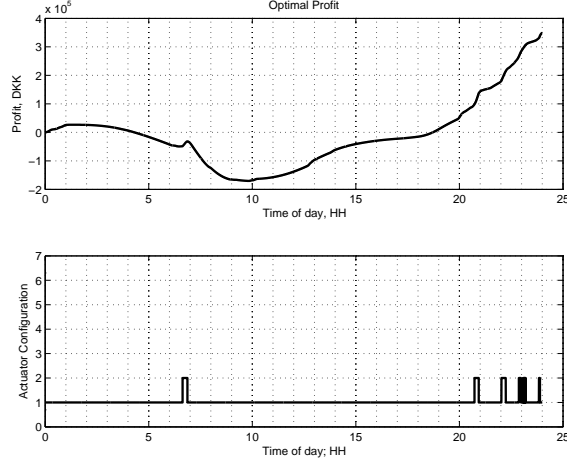


Figure 5.14: Profit and optimal fuel configuration over 24 hours of operation during June 29th, 2008 with 25% production capabilities of gas and oil.

signal to the valves controlling these flows is denoted $\mathbf{u} = (u_c, u_g, u_o) \in U$ and the system equations are given by

$$\begin{aligned} \dot{\mathbf{z}}(t) &= \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t), \\ \mathbf{x}(t) &= \mathbf{C}\mathbf{z}(t), \end{aligned} \quad (5.18)$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{A}_c & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{A}_g & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{A}_o \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_{i1} & k_{i2} & k_{i3} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \mathbf{B}_c & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{B}_g & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{B}_o \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ k_{i0} \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} \mathbf{C}_1 & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{C}_1 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{C}_1 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \end{aligned}$$

and $k_{ij}, i \in \mathcal{I}$, are constants describing the dynamics of the three fuel systems which are obtained from transfer functions of the form $H_i(s) = (\tau_i s + 1)^{-3}$ where $\tau_i, i \in \mathcal{I}$, is 90, 60, and 70, respectively. In the sequel the control set U is assumed compact and convex.

Moreover the function

$$h(\mathbf{z}, t) = \Upsilon \mathbf{z} + \psi(t), \quad (5.19)$$

is introduced with

$$\begin{aligned} \mathbf{r} &= \begin{bmatrix} \gamma^T \mathbf{Q} \\ -\gamma^T \mathbf{Q} \end{bmatrix}, \\ \psi(t) &= \begin{bmatrix} \gamma^T \mathbf{b} - y_r(t) + \alpha \\ -\gamma^T \mathbf{b} + y_r(t) + \alpha \end{bmatrix}. \end{aligned}$$

Hence h is constructed such that the set $Z' = \{(z, t) \mid h(z, t) \geq 0\}$ determines a “reference band” around the reference, $y_r(t)$. Here α should be thought of as a parameter dictating the size of the reference band.

In the sequel the map g_P , defined by (5.14), needs to be continuous. To obtain this it is assumed that the non continuous contributions, i.e. the maps h_i defined by (5.3), are replaced by continuous approximations. The obtained map will, by abuse of notation, also be denoted by g_P

Combining the above the optimization problem is formulated as

$$\max_{(z(t), u(t)) \in \Omega} \int_0^T g_P(\mathbf{C}z(t), t) dt, \quad (5.20)$$

subject to

$$\dot{\mathbf{z}}(t) = \mathbf{A}z(t) + \mathbf{B}u(t), \quad 0 \leq t \leq T, \quad (5.21)$$

$$u(t) \in U, \quad 0 \leq t \leq T, \quad (5.22)$$

$$h(\mathbf{z}(t), t) \geq 0, \quad 0 \leq t \leq T, \quad (5.23)$$

where Ω is the set of admissible⁷ pairs $(z(t), u(t))$. Note that by choosing the control set U and parameter α in (5.19) appropriate Ω is assumed non empty.

Now since (the reference band) Z' is compact and the set

$$Q(z, t) = \{(s, \mathbf{q}) \mid s \geq g_P(z, t), \mathbf{q} = \mathbf{A}z + \mathbf{B}u, u \in U\},$$

is convex for every $(z, t) \in Z'$, the Filippov Existence Theorem (see [12, p. 199]) may be used to conclude that the above optimization problem has an absolute maximum in Ω .

The approach described above will be studied in detail in future papers. In particular we remark that some results have been obtained in the paper [13] where linear programming is used to solve the problem. This is obtained by approximating g_P by a piece-wise affine function and converting the dynamics, profit function, and constraint function in to discrete time.

9 Discussion

In this work models of two of DONG Energy’s business objectives (Efficiency and Controllability) have been formulated such that a selection between three different fuels can be performed in an optimal manner. Profit maximization is considered as a optimality measure as this is an important measure for companies today.

⁷That is $z(t)$ is absolutely continuous, $u(t)$ is (Lebesgue) measurable and $z(t), u(t)$ satisfying (5.21), (5.22) and (5.23).

A static modelling and optimization is performed such that the optimal configuration can be found for a given production setpoint. The developed optimization method is then expanded to handle changes in prices and production reference. The result from this expansion is compared to a case where only coal is present and the use multiple fuels does increase the profit by 12% over 24 hours of operation.

How the result is affected by a reduction of 75% in the gas and oil system is, furthermore, examined. The gain of mixing the fuels is reduced, however, during 24 hours of operation the difference in profit compared to only using coal is 8%.

The result from this work can be used in two way; online to determine which fuels to use during the day and offline to determine if a plant could be instrumented with additional fuels such that the profit is increased.

An extension to fault detection could be relevant as this works could be used online in combination with fault detection methods [14]. Two possible scenarios are relevant depending on the seriousness of the detected fault; rerun the planning to optimize the profit given the new conditions or schedule maintenance during periods the failed actuator system is not in use.

Furthermore, with the changes in the demand for environmental friendly energy the current electric market is going to change dramatically during the next couple of years where more renewable energy will come into play. As many of the renewable energy systems are dependent of the forces of nature, the use of decentral short-time storage of energy will increase (e.g. electric cars [15] and [16]). These short-time storage sources could be seen as an additional actuator in the methods presented in this work and thus planing for the entire electrical grid (in some region) is a possible extension of this work. Similar, work in this direction has been seen in [17] for Norwegian hydro-power plants.

10 Acknowledgement

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Paper C

Optimal Production Planning of a Power Plant

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Abstract

This paper addresses the problem of planning the usage of actuators optimally in an economic perspective. The objective is to maximize the profit of operating a given plant during 24 hours of operation. Models of two business objectives are formulated in terms of system states and the monetary value of these objectives is established. Based on these and the cost of using the different actuators a profit function has been formulated. The optimization of the profit is formulated as an optimal control problem where the constraints include the dynamics of the plant as well as a requirement to reference tracking. A power plant is considered in this paper, where the fuel system consists of three different fuels; coal, gas, and oil.

1 Introduction

The requirements for a complex process control system are usually derived from a top level (business) requirement to the entire system which is to maximize the income or profit of a company. However, the requirements specification for a process control system rarely includes profit maximization directly. Instead the designer works with requirements on settling time, rise time, bandwidth, disturbance rejection and so on, because these are easy to evaluate through simulation and are well defined with respect to transfer functions and the pole placement of the closed loop system. All of these measures assume that a set of actuators and sensors is given. The choice of this set of actuators and sensors does, however, influence the operating cost and performance of the system greatly - this will be addressed in this paper.

The economical cost of instrumenting a plant with sensors and actuators has, on the other hand, been considered in the selection method presented in [1], where the precision of a sensor or an actuator is assumed to be proportional to its cost. By introducing a bound on the economical cost of the instrumentation it is possible to formulate the design problem as convex optimization. This helps the designer to select the right instrumentation. However, this method only considers the implementation cost and not the operational cost which in many cases is the main concern for minimization [2].

As the requirements for a process control system usually are derived from business objectives it would be natural to include these business objectives when configuring the sensor/actuator layout of a plant. An attempt of this has been presented in [3] where functionals describing the business objectives are maximized. The functionals have been established using data from nordpool¹ which is a marketplace for trading power contracts. This marketplace has also been used by [4] where the control of water resources in Norway is considered. An optimization of how to use different hydro plants is performed on basis of market prices and commitments.

This work will extend the work in [5] where notions from production economics have been used to formulate the objectives of a Danish power plant company. The outputs of the system are measures of the business objectives and the input is the flow of fuels. The optimization performed in [5] does not consider the dynamics of the plant and assumes that it is possible to switch from one fuel to another instantaneously.

¹www.nordpool.dk

In this work the dynamics of the fuel systems are included in the optimization and it is shown that the dynamics influence the gain in profit. Our result is a production strategy which maximizes the profit during 24 hours of operation.

1.1 Outline

A description of the problem considered in this paper is presented in Section 2 and the relevant models are then developed in Section 3. These include the time varying parameters, the dynamics of the plant, and measures of the business objectives. In Section 4 the problem is stated in mathematical terms as an optimal control problem. The optimal control problem is discretized using zero-order hold sampling and the resulting optimization problem is approximated by a linear program. The numerical results are presented in Section 5 and some final remarks are made in Section 6.

2 Problem Description

The problem in this work has been formulated in collaboration with DONG Energy - a Danish power provider. The goal of any company is to maximize its profit and for DONG Energy the profit maximization has been divided into four individual business objectives; efficiency, controllability, availability, and life time. However, to simplify the model, only the two first objectives are considered in this work. The problem formulation is based on a model of a coal fired boiler - a vital component of a power plant - which is augmented with two additional fuel systems; gas and oil. The three different fuels have certain advantages and disadvantages e.g. gas is easy to control but is an expensive fuel. Some of the characteristics of the different fuels are:

Coal is advantageous when considering the price per stored energy, however, it is difficult to control as unmeasurable fluctuations in the coal flow are introduced by the coal mill when the coal is ground to coal dust. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind the coal which needs to be considered.

Gas is more expensive than coal and energy is not converted to steam as efficiently with gas as with coal due to the layout of the chosen boiler. However, gas arrives at the power plant under high pressure which is lowered using a turbine generating electrical energy. Furthermore, gas is much easier to control as it is possible to measure the flow.

Oil is, with the current market prices, the most expensive of the three fuels and has to be heated before entering the boiler. This process demands energy itself. Nevertheless, oil is considered in this work as it is possible to measure the oil flow into the boiler and this makes it easy to control. Furthermore, oil is present in most existing coal fired plants as oil is used to start up the plant.

The focus of this work is to derive a plan for optimal usage of the three fuels described above during 24 hours of operation. Optimal usage is defined as maximizing the profit in terms of the two considered objectives; efficiency and controllability. Efficiency is a measure of how efficient a fuel is converted into electricity and controllability is a measure

of the plant's capability to change the production level. Furthermore, the production level of the plant should follow a time varying reference as closely as possible.

3 Plant Model

Due to changes in demand the electricity production of a power plant is not constant during the year or even during 24 hours. It is, however, possible to make a prediction of the demands in the future and each power plant therefore knows the expected production plan 24 hours ahead. Besides the production plan the prices of electricity and controllability are also known in advance. Using these three parameters, and how they change, it is possible to plan the usage of fuels. In the following a description is given of how the prices and production changes (a description of the planning can be found in [6]).

3.1 Production Plan

An example of a production plan for the considered plant is depicted in Figure 6.1. The graph depicts the production from midnight the 29th of June, 2008 and 24 hours forward. As seen in the figure the production is low during the night but at 6:00-7:00 in the morning there is a steep gradient caused by the increase in consumption when people and companies start to use electricity. The production plan is modelled as an smooth approximation

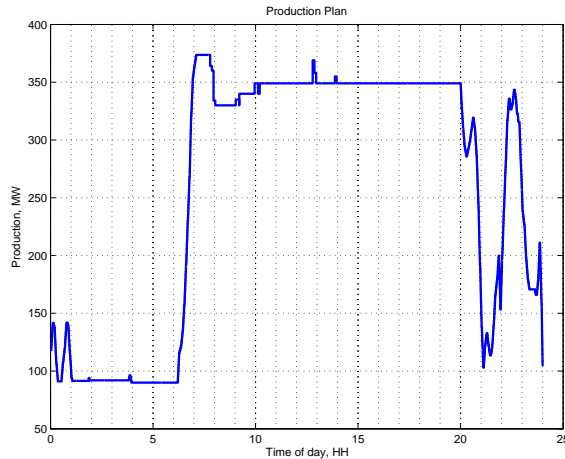


Figure 6.1: The production during June 29th, 2008. The data used to generate this plot has been provided by DONG Energy.

of the graph depicted in Figure 6.1 and is denoted by²

$$t \mapsto y_r(t) \quad [MW]. \quad (6.1)$$

The smoothness assumption is purely theoretical (see (6.3)). In simulation the production plan (6.1) will be replaced by the non-smooth function defined by the graph in Figure 6.1.

²[.] indicates the units and in this case $y_r(t)$ is measured in Mega Watt.

3.2 Efficiency Price

The price of electricity, p_{R1} , changes during the day as the demand changes, i.e., during the middle of the day when the demand is greatest the price is also higher than during the early morning. The electricity price from the 29th of June 2008 is depicted in Figure 6.2. In this work³

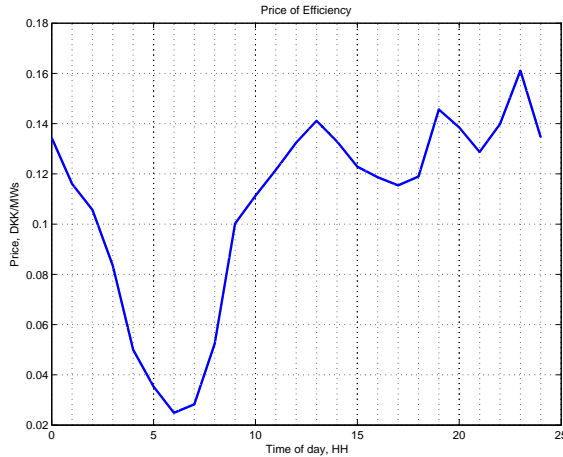


Figure 6.2: The electricity price during June 29th, 2008. The data used to generate this plot has been found on www.nordpool.dk

$$t \mapsto p_{R1}(t) \quad [DKK/MWs]. \quad (6.2)$$

denote the efficiency price defined by the graph in Figure 6.2.

3.3 Controllability Price

Large gradients in the production plan, as seen in Figure 6.1 around 6:00-7:00, yield a high price on controllability as it is likely that some plants are not capable of generating the gradients needed. In general this would be related to the derivative of the production plan and thus the price is higher during the periods in the morning and afternoon/evening where there exists steep gradients in Figure 6.1. The controllability price is defined as

$$t \mapsto p_{R2}(t) = \beta \left| \frac{d}{dt} y_r(t) \right|, \quad [DKK/MW], \quad (6.3)$$

where $\beta = 1000$ is a factor which has been determined in collaboration with DONG Energy.

In the simulations the differential quotient in (6.3) is replaced by a difference due to the non-smooth properties of (6.1). The resulting graph of the simulated version of (6.3) is depicted in Figure 6.3.

³DKK is the Danish currency, kroner.

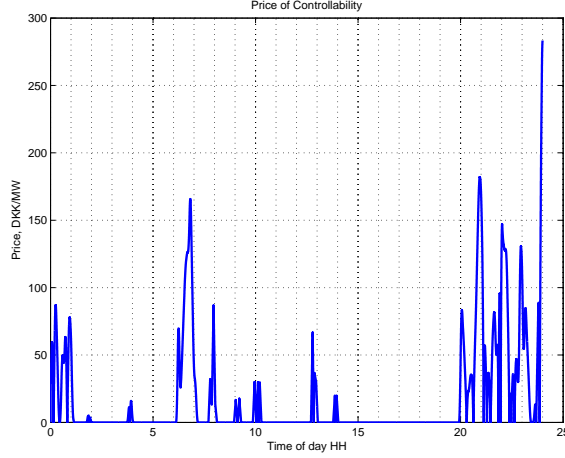


Figure 6.3: Modelled controllability price during the 29th of June, 2008. The data in this plot has been established in collaboration with DONG Energy.

3.4 Fuel Price

Obviously the fuel prices change over time, however, these changes are slow compared to the changes in the efficiency and controllability prices as the time span is a matter of weeks. Therefore, the fuel prices are considered as constants and the fuel prices are given as

$$\mathbf{p}_C = (p_{C1}, p_{C2}, p_{C3}) = (1.20, 3.74, 6.00) \quad (6.4)$$

with unit in $[DKK/kg]$.

3.5 Input-Output Mapping

Let \mathbb{R}_+^3 denote the set of positive elements in \mathbb{R}^3 , i.e., $\mathbb{R}_+^3 = \{\mathbf{v} \in \mathbb{R}^3 | \mathbf{v} \geq 0\}$ where the inequality is to be understood coordinate-wise (this notation will be used throughout this work). The input space, U , and the flow space, X , are now given by

$$\begin{aligned} U &= \{\mathbf{v} \in \mathbb{R}_+^3 | 0 \leq \mathbf{v}^T \mathbf{e}_u \leq c_u\}, \\ X &= \{\mathbf{v} \in \mathbb{R}_+^3 | 0 \leq \mathbf{v}^T \mathbf{e}_x \leq c_x\}, \end{aligned} \quad (6.5)$$

where the vector $\mathbf{e}_j = (e_{j1}, e_{j2}, e_{j3}) \in \mathbb{R}^3$ with $e_j > 0$ and scalar $c_j \in \mathbb{R}$ for $j \in \{u, x\}$ are to be determined later where their physical interpretation also will be given. Note that U (resp. X) is the 3-simplex in \mathbb{R}_+^3 with vertices $\mathbf{0}$, $(c_u/e_{u1}, 0, 0)$, $(0, c_u/e_{u2}, 0)$, and $(0, 0, c_u/e_{u3})$, (resp. $\mathbf{0}$, $(c_x/e_{x1}, 0, 0)$, $(0, c_x/e_{x2}, 0)$, and $(0, 0, c_x/e_{x3})$). Each (flow) state

$$\mathbf{x} = (x_c, x_g, x_o) \in X, \quad ([kg/s], [kg/s], [kg/s]),$$

in the system describe the flow of coal, gas, and oil, respectively. In the sequel we let $\mathcal{I} = \{c, g, o\}$ where the elements of the index set \mathcal{I} refers to the three different fuels. Occasionally the identification $(c, g, o) = (1, 2, 3)$ will be used.

The output space $Y = Y_1 \times Y_2$ is a subset of \mathbb{R}^2 where each output

$$\mathbf{y} = (y_e, y_c) \in Y, \quad ([MW], [MW/s]),$$

of the system describe the two objectives; efficiency and controllability, respectively. Both of these quantities contain contributions from coal, gas, and oil as they will be defined as functions of the fuels later.

Furthermore, a state space, Z , is defined as

$$Z = \{z = (z_1, z_2, \dots, z_9) \in \mathbb{R}^9 | (z_1, z_4, z_7) \in X\},$$

which is used when describing the dynamics of the fuel flows.

Plant Dynamics

The fuel flow, $\mathbf{x}(t)$, into the power plant is governed by third order differential equations (these equations also include the power plant dynamics). The control signal to the valves controlling these flows is denoted $\mathbf{u} = (u_c, u_g, u_o) \in U$ and the dynamics is given by

$$\begin{aligned} \dot{\mathbf{z}}(t) &= \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{x}(t) &= \mathbf{C}\mathbf{z}(t), \end{aligned} \tag{6.6}$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{A}_c & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{A}_g & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{A}_o \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ h_{i1} & h_{i2} & h_{i3} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \mathbf{B}_c & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{B}_g & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{B}_o \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ h_{i0} \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} \mathbf{C}_1 & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{C}_1 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{C}_1 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \end{aligned}$$

and $h_{ij}, i \in \mathcal{I}$, are constants describing the dynamics of the three fuel systems which are obtained from transfer functions of the form

$$H_i(s) = \frac{1}{(\tau_i s + 1)^3},$$

where $\tau_i, i \in \mathcal{I}$, is 90, 60, and 70, respectively. The three fuel systems may have some shared dynamics but to simplify the model in this work the systems are assumed decoupled.

Functions describing the two business objectives are derived in the following.

Efficiency

The efficiency objective, $y_e = y_e(\mathbf{z})$, deals with how much electricity is produced from a certain amount of fuel. Three affine functions describing the contribution of the individual

fuels to the efficiency objective have been established using measurement data from two Danish power plants and can be expressed as

$$\tilde{y}_e(z) = Qz + b, \quad (6.7)$$

where

$$Q = \text{diag}(e_x)C, \quad e_x = (10.77, 18.87, 15.77), \\ b = (-1.76, 1.85, -0.37),$$

and C defined in (6.6). The values of e_x and b have been established using measurement data and are measured in $[MJ/kg]$ and $[MW]$ respectively. The energy used for preprocessing the individual fuels is expressed by the b_i 's and the e_{x_i} 's are conversion factors which are a combination of the boiler efficiency and energy storage in the different fuels. Note the constant e_x in (6.5) is now defined.

The total amount of efficiency is described by the function

$$Z \rightarrow Y_1; z \mapsto y_e(z) = \gamma^T \tilde{y}_e(z),$$

where

$$\gamma = (1, 1, 1).$$

The constant c_x in (6.5), can now be determined by $c_x = 400 - \gamma^T b$, where 400 refers to the maximum efficiency (in $[MW]$) produced by the plant and $\gamma^T b$ is the total own-consumption of the plant used for preprocessing the three fuels. We let $c_u = c_x$ and $e_u = e_x$ in (6.5) since (6.6) has negative real eigenvalues and the steady state gain is 1 which guarantees that $x(t) \in X$ during any steady state operation.

Controllability

The controllability objective, $y_c = y_c(z)$, deals with a measure of how fast the production of electricity can be changed. Allowed changes in the production is limited to a certain gradient depending on the current efficiency. The reason for this limit is a compliance to maximum temperature gradients in the boiler (these have not been explicitly modelled and are therefore indirectly considered by limiting the allowed changes). When using coal it is allowed to change production with $0.133 [MW/s]$ when running the plant at low and high production and $0.267 [MW/s]$ in the middle range from $200 [MW]$ to $360 [MW]$. When using oil or gas the values are $0.133 [MW/s]$ and $0.534 [MW/s]$. If a mixture of the three fuels are used it is assumed that the allowed change is a linear combination of the allowed change of the individual fuels. The controllability objective is, therefore, modelled as

$$Z \rightarrow Y_2; z \mapsto y_c(z) = \begin{cases} 0.133 & y_e(z) \in S_1 \\ \frac{\xi^T \tilde{y}_e(z)}{y_e(z)} & y_e(z) \in S_2 \\ 0.133 & y_e(z) \in S_3, \end{cases} \quad (6.8)$$

where

$$\xi = (0.267, 0.534, 0.534), \quad S_1 = \{s \in \mathbb{R} | 0 \leq s \leq 200\}, \\ S_2 = \{s \in \mathbb{R} | 200 < s < 360\}, \text{ and} \\ S_3 = \{s \in \mathbb{R} | 360 \leq s \leq 400\}.$$

3.6 Prices

The cost of using the fuel, \mathbf{x} , revenue from production of output, \mathbf{y} , and the profit of operating the power plant can now be determined. The above constructions yields a product (or output) function, y_P , of the system given by

$$y_P : Z \rightarrow Y; \mathbf{z} \mapsto (y_e(\mathbf{z}), y_c(\mathbf{z})).$$

The growth of cost and growth of revenue for the system are defined by the following functions (both with units in [DKK/s])

$$\begin{aligned} g_C : Z \rightarrow \mathbb{R}; \mathbf{z} &\mapsto \mathbf{z}^T \mathbf{C}^T \mathbf{p}_C, \\ g_R : Y \times \mathbb{R}_+ &\rightarrow \mathbb{R}; (\mathbf{y}, t) \mapsto \mathbf{y}^T \mathbf{p}_R(t), \quad \mathbf{p}_R(t) > 0, \end{aligned}$$

where \mathbf{p}_C is as defined in (6.4) and

$$\mathbf{p}_R(t) = (p_{R1}(t), p_{R2}(t))$$

with the coordinate functions as defined in (6.2) and (6.3).⁴

The growth of profit is hence defined by

$$Z \times Y \times \mathbb{R}_+ \rightarrow \mathbb{R}; (\mathbf{z}, \mathbf{y}, t) \mapsto g_R(\mathbf{y}, t) - g_C(\mathbf{z}),$$

which for the system yields the function

$$g_P : Z \times \mathbb{R}_+ \rightarrow \mathbb{R}; (\mathbf{z}, t) \mapsto g_R(\mathbf{y}_P(\mathbf{z}), t) - g_C(\mathbf{z}).$$

Therefore, the profit is given by

$$P : \mathbb{R}_+ \rightarrow \mathbb{R}; t \mapsto \int_0^t g_P(\mathbf{z}(\tau), \tau) d\tau. \quad (6.9)$$

4 Optimization

The objective of the company is to maximize its profit over the planning horizon, T , such that the production plan is fulfilled with the available fuel systems. This optimization is stated as

$$\begin{aligned} &\max_{u \in U} P(T) \\ &\text{subject to} \\ &\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u}, \\ &h(\mathbf{z}(t), t) = \Upsilon \mathbf{z}(t) + \psi(t) \geq 0, \end{aligned} \quad (6.10)$$

where

$$\Upsilon = \begin{bmatrix} \gamma^T \mathbf{Q} \\ -\gamma^T \mathbf{Q} \end{bmatrix}, \quad \psi(t) = \begin{bmatrix} \gamma^T \mathbf{b} - y_r(t) + \alpha \\ -\gamma^T \mathbf{b} + y_r(t) + \alpha \end{bmatrix}.$$

⁴The prices used in this work corresponds to the market prices the 29th of June, 2008 and has been established using internal DONG Energy documents and the archive of power price at www.nordpool.dk, which is a marketplace for trading power contracts.

Hence the function $h(z(t), t)$ is constructed such that the efficiency, $y_e(z)$, follows the production plan, $y_r(t)$, within a bound α . We have omitted the constraint on $x(t)$, i.e., $x(t) \in X$. It is easy to include in the optimization but here we have decided to just verified this a posteriori.

The growth of profit function can be simplified when the reference is followed perfectly, i.e., $\alpha = 0$. Then $y_e(z(t)) = y_r(t)$ which yields

$$g_P(z(t), t) = \Theta(t)z(t) + \varphi(t), \quad (6.11)$$

where

$$\begin{aligned} \Theta(t) &= p_{R1}(t)\gamma^T Q - p_C^T C + p_{R2}\vartheta(t), \\ \varphi(t) &= p_{R1}(t)\gamma^T b + p_{R2}\zeta(t), \end{aligned}$$

and $\vartheta(t)$ and $\zeta(t)$ makes up for the switching function in (6.8), i.e.,

$$\vartheta(t) = \begin{cases} 0 & y_r(t) \in S_1 \\ \frac{\xi^T Q}{y_r(t)} & y_r(t) \in S_2 \\ 0 & y_r(t) \in S_3 \end{cases}, \quad \zeta(t) = \begin{cases} 0.133 & y_r(t) \in S_1 \\ \frac{\xi^T b}{y_r(t)} & y_r(t) \in S_2 \\ 0.133 & y_r(t) \in S_3 \end{cases},$$

with the functions, constants, and sets as previously defined. The assumption $\alpha = 0$, might not be feasible because the demand might change quicker than what is possible with the dynamics of the fuel systems. However, (6.11) will be used as an approximation for the real $g_P(z(t))$ when $\alpha \neq 0$.

4.1 Discrete optimization

In this section the cost, constraint, and system from the previous section will be converted into discrete time. From the discrete time cost, constraint, and system a linear program formulation of the problem will be obtained.

First, however, some assumptions about the problem will be made. The time period T is divided into N equally sized time units, h , i.e., $T = Nh$. It is assumed that $\Theta(t)$, $\varphi(t)$, and $\psi(t)$ can be approximated by piecewise constant functions for each time step, i.e.,

$$\begin{aligned} \Theta(t) &= \Theta_k, & kh < t < (k+1)h, \\ \varphi(t) &= \varphi_k, & kh < t < (k+1)h, \\ \psi(t) &= \psi_k, & kh < t < (k+1)h. \end{aligned}$$

Furthermore, the control will be assumed piecewise constant as customary when digital to analogue conversion is performed using sample-and-hold circuits.

Using a fact from [7] the continuous time state $z(t)$ in the dynamic system in (6.6) can be described by

$$\begin{aligned} z(t) &= e^{At} z_0 + \int_0^t e^{A(t-s)} B u_0(s) ds \\ &= \begin{bmatrix} I & 0 \end{bmatrix} \exp \left\{ \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} t \right\} \begin{bmatrix} z_0 \\ u_0 \end{bmatrix}, \end{aligned} \quad (6.12)$$

where \mathbf{I} is an identity matrix with appropriate dimension. Using (6.12) it is possible to derive the following formula which is used during the discretization of the cost and constraint.

$$\begin{aligned} \int_0^h e^{\mathbf{A}t} dt &= e^{\mathbf{A}h} \int_0^h e^{-\mathbf{A}(h-t)} dt \\ &= e^{\mathbf{A}h} \left(e^{-\mathbf{A}h} \cdot 0 + \int_0^h e^{-\mathbf{A}(h-t)} \mathbf{I} dt \right) \\ &= e^{\mathbf{A}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \exp \left\{ \begin{bmatrix} -\mathbf{A} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} h \right\} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \end{aligned} \quad (6.13)$$

4.2 System

The system equation is sampled forming the well known discrete system equations

$$\mathbf{z}_{k+1} = \Phi \mathbf{z}_k + \Gamma \mathbf{u}_k,$$

where

$$\Phi = e^{\mathbf{A}(t_{k+1}-t_k)} \text{ and } \Gamma = \int_0^{t_{k+1}-t_k} e^{\mathbf{A}s} ds \mathbf{B}.$$

4.3 Cost

When deriving a sampled version of the cost the integral is split into a sum of N integrals and then (6.12) and (6.13) are used to derive a discrete cost function, i.e.,

$$\begin{aligned} P(T) &= \sum_{k=0}^{N-1} \int_{kh}^{(k+1)h} (\Theta(t) \mathbf{z}(t) + \varphi(t)) dt \\ &= \sum_{k=0}^{N-1} \Theta_k \int_0^h \left(e^{\mathbf{A}t} \mathbf{z}_k + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B} ds \mathbf{u}_k \right) dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \Theta_k \int_0^h \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}t} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} + h\varphi_k \\ &= \sum_{k=0}^{N-1} \mathbf{C}_k \mathbf{z}_k + \mathbf{D}_k \mathbf{u}_k + \mathbf{E}_k, \end{aligned}$$

where

$$\begin{aligned} \mathbf{C}_k &= \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \\ \mathbf{D}_k &= \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \\ \mathbf{E}_k &= h\varphi_k, \quad \hat{\mathbf{A}} = \begin{bmatrix} -\tilde{\mathbf{A}} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \end{aligned}$$

and \mathbf{I} denoting identity matrices of appropriate dimensions.

4.4 Constraint

The constraint needs to be satisfied at all times which, of course, is not guaranteed by ensuring the constraint is satisfied at each sample time. In order to approximate this, the constraint is sampled L times between each sample of the cost. The discrete version of the constraint is described by

$$\begin{aligned} h(\mathbf{z}(t), t) &= \Upsilon \mathbf{z}(t) + \psi(t) \\ &= \Upsilon \left(e^{\mathbf{A} \frac{1}{L} h} \mathbf{z}_k + \int_0^{\frac{1}{L} h} e^{\mathbf{A}(\frac{1}{L} h - s)} \mathbf{B} ds \mathbf{u}_k \right) + \psi\left(\frac{l}{L} h + kh\right) \\ &= \Psi_l \mathbf{z}_k + \Pi_l \mathbf{u}_k + \Omega_{k,l}, \end{aligned}$$

where

$$\begin{aligned} \Psi_l &= \Upsilon e^{\mathbf{A} \frac{1}{L} h}, \quad \Pi_l = \Upsilon \int_0^{\frac{1}{L} h} e^{\mathbf{A}(\frac{1}{L} h - s)} \mathbf{B} ds, \text{ and} \\ \Omega_{k,l} &= \psi\left(\frac{l}{L} h + kh\right). \end{aligned}$$

Now, the problem in (6.10) can be approximated by a linear program where the constraint is not guaranteed to be satisfied at all times but it is, however, satisfied at LN equally spaced points in time. Furthermore, the cost function is approximated by (6.11) which is a good approximation when α is small. To ensure this α is made time-varying and the cost function is augmented with an α -term (and appropriate weight W_k). The linear program can thus be stated as

$$\begin{aligned} &\max_{\substack{\mathbf{u} \in U \\ \alpha \geq 0}} \sum_{k=0}^{N-1} (\mathbf{C}_k \mathbf{z}_k + \mathbf{D}_k \mathbf{u}_k + \mathbf{E}_k - W_k \alpha_k) \\ \text{subject to} \quad &\mathbf{z}_{k+1} = \Phi \mathbf{z}_k + \Gamma \mathbf{u}_k, \\ &\Psi_l \mathbf{z}_k + \Pi_l \mathbf{u}_k + \Omega_{k,l} \geq 0. \end{aligned}$$

5 Results

The linear program stated in the previous section has been formulated using YALMIP [8] and solved using SeDuMi⁵. In this section the results will be presented where the following values have been used

$$T = 86400s, \quad N = 432, \quad h = 200s, \quad L = 5, \quad W_k = \frac{500000}{NL}.$$

Figure 6.4 depicts α vs time which shows that the approximation of the cost function is good as the values are below 14 at all times, which is within 3.5% of full production (and less at most times).

⁵SeDuMi is a software package used to solve optimization problems (see <http://sedumi.ie.lehigh.edu/content/view/17/53/>)

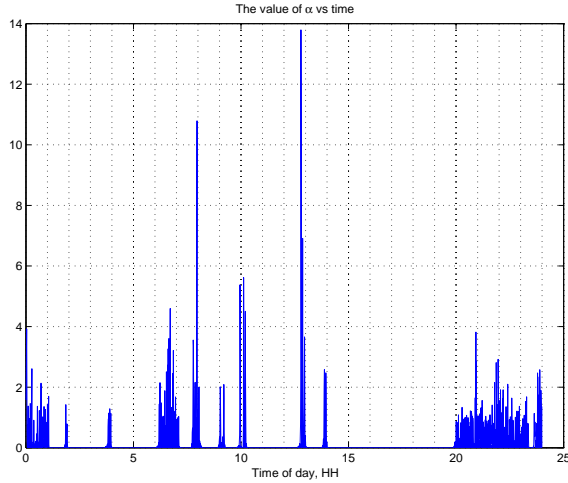


Figure 6.4: The efficiency is also equal to the production plan reference at all times as α is small.

The profit over time, $P(t)$, is depicted in Figure 6.5 and is low during most of the morning. Actually, from approximately 2:00-10:00 the gain in profit is negative which is caused by the low price on efficiency, p_{R1} . At 10:00 the profit starts to grow and at the end of the day the total profit is approximately 330000DKK.

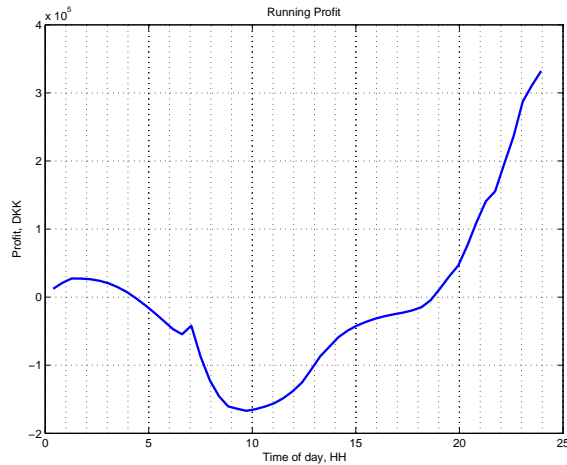


Figure 6.5: Optimal profit during 24 hours of operation June 29th, 2008.

The usage of the three fuel systems is illustrated in Figure 6.6, where the input signals to the coal, gas, and oil systems are depicted. Coal is used as the primary fuel during the day, but at times the gas system is used to compensate for the slow coal system during transients. This can especially be observed around 6:00-7:00 and at the evening.

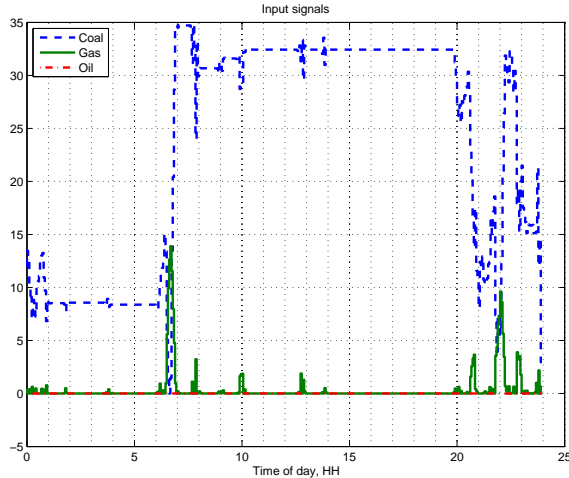


Figure 6.6: The input signal to the fuel systems shows that only coal and gas is used.

6 Discussion

Comparing the results of this work with the results from [5], where no dynamics were present, it can, as expected, be concluded that the dynamics should be considered as the profit is different. However, the usage of fuels are comparable as gas is used during periods with large gradients in the reference. The profit found in Section 4 is smaller than the profit obtained without dynamics in [5], but it also greater than the profit obtained when running the plan only with coal. Thus, mixing fuels is beneficial under consideration of the two business objectives presented in this paper.

Furthermore, the usage of the fuels does not switch completely from one fuel to another and thus the gas and oil systems are not fully used - the oil system is actually not used at all. This would suggest that a new plant should only be instrumented with a full coal system and a partial gas system.

Future work could include expanding the business models to include more detail about the bidding and settling of prices performed at Nordpool. In particular, the controllability model and price have been simplified in this work.

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Paper D

Profit Maximization of a Power Plant

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Abstract

This paper addresses the problem of profit maximization of a power plant by utilizing three different fuel systems in an optimal manner. Pontryagin's maximum principle is used to derive properties of the optimal control strategy. These properties give rise to a switching function. Subsequently, certain heuristics are introduced and used in combination with discrete optimization to obtain an initial trajectory of the switch function. An iterative procedure is proposed which uses the initial trajectory for the computation of the optimal control strategy. The control strategy derived is a combination of a state feedback and time-varying feedforward term. Its performance is tested against input noise.

1 Introduction

The economic perspective is rarely considered when developing control structures and strategies for process control systems. Indeed, requirements are most frequently imposed on disturbance rejection, pole placement or other well known system theoretic properties. Nonetheless, the economics of control has gained some focus for example the selection of sensor and actuator [1] and the design of controller structure [2]. Furthermore, optimal steady state operations have been studied [3].

In [4], the hydro power production in Norway is considered by maximizing the profit of a hydro plant such that the production commitment of the current day is fulfilled. The electrical market is considered in the optimization and planning of the production for which stochastic programming is used.

The work in this paper is similar to the work in [4] as profit maximization in electricity production is considered. However, a traditional power plant will be considered, and optimal control will be used for the profit optimization ([5, 6]).

A problem of the optimal operation of a power plant in a liberal electricity market is a subject of [7]. In this work, two types of power plants are considered: a hydro-electric and a thermal power plant. The main focus is on the dynamic modeling of electricity-production and the price of electricity. Both of them are described by random processes. As a consequence, the suggested approach to the optimal operation of a power plant is stochastic optimization, which is formulated in terms of nonlinear partial-integro-differential equations. To solve them, the authors use a finite difference scheme.

In like manner, [8] addresses the problem of balancing the power on electricity market consisting of wind energy and hydropower. Also in this work, the demand for balancing power and the electricity price are described by stochastic models. Subsequently, the hydropower-scheduling of trading decisions are formulated as a stochastic optimization problem. To solve it, the stochastic variables are approximated by a finite set of scenarios, so called scenario-trees. Afterward, the optimization problem is solved by means of stochastic programming.

A power plant capable of using a number of different fuels is considered in this paper. The fuels of interest are coal, gas, and oil. They certainly have certain advantages and disadvantages, e.g., coal is an inexpensive fuel, but it is difficult to control. The objective of this work is to maximize the profit of the power plant when following a predefined production reference.

The problem stated above has been discussed in [9], [10], [11], and [12]. In particular, the formulation in [12] which includes plant dynamics gives the basis of this work. In the previous works, a function for the instantaneous profit flow has been determined, which includes time-varying measures of business objectives and time varying price data obtained from Nord Pool¹. The objective is to maximize the integral of the instantaneous profit flow over time, i.e., maximizing the profit of the company. The requirement of following a predefined reference has been formulated as a side-constraint in the optimization, which has been solved in discrete time. Whereas, in this work the tracking will be included as a penalty term in the objective function. This yields a simpler problem, which substantially reduces the computation time.

In this work, a continuous control strategy is given that maximizes the profit of a power plant. The strategy is obtained by using Pontryagin's maximum principle to devise some properties of the optimal control input. The optimal solution consists primarily of singular arcs which is known to make the optimal control problem more difficult to solve numerically. In this work, we propose an approach that results in a combined feedback and feedforward solution, which yields a profit 30% greater than using an input signal obtained by discrete optimization.

The results in this paper could be of interest in a model predictive control (MPC) context. Indeed, if the optimal solution is known to consist of singular arcs then this information should be used. By computing the feedback law that generates the singular arcs better performance and lower sampling rate may be achieved. This should be particularly useful when the model and the data is provided in continuous time over a large time horizon.

1.1 Outline

A model of the plant considered in this work is presented in Section 2. Furthermore, the models of the business objectives and optimization problem are presented here. In Section 3, Pontryagin's maximum principle is applied to the optimization problem and some properties of the optimal input are derived. As the optimal input is dependent on an unknown switch function, the optimization problem is converted to discrete time, and subsequently, an estimate of the switch function is computed. This procedure is carried out in Section 4. The switch function is applied to the optimal input strategy in Section 5, and the resulting profit is compared to what was possible with the discrete optimization. In Section 6, the control strategy is evaluated when input noise is present, and finally, in Section 7, a discussion of the results is given. Furthermore, two appendices are included where the optimal continuous control strategy and a discrete version of the objective function are given.

2 Problem Formulation

In this section the models from our previous work will be recalled and then the optimization problem will be presented. First, an introduction to the considered plant is given.

The problem considered in this work is based on a coal fired boiler power plant which is depicted in Figure 7.1 and consists of the following components:

¹Nord Pool is the Nordic electrical market, where power contracts are traded.

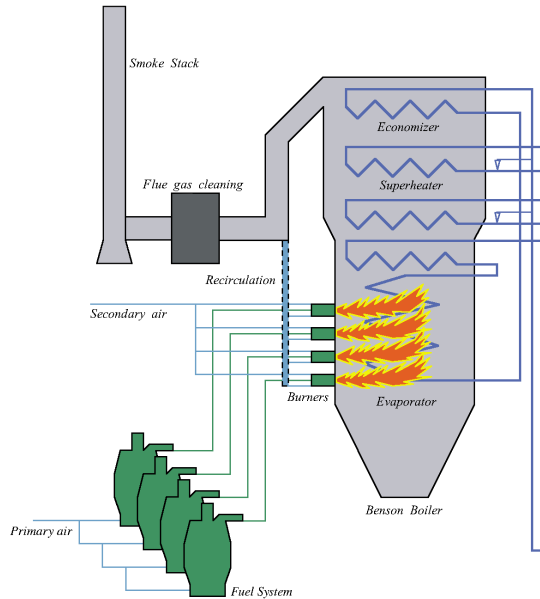


Figure 7.1: Benson boiler model.

Coal mills The coal mills grind the coal to small dust particles which burn quickly and efficiently. However, it is difficult to control the amount of dust the coal mills deliver as it is not possible to measure the dust flow into the furnace.

Furnace The furnace is a module where the coal dust (or other fuels) is burned; thereby, heat is delivered to the boiler.

Evaporator The evaporator is fed with water, which is evaporated under high pressure by the heat from the burners.

Superheater The superheater (super) heats the steam from the evaporator.

Economizer The economizer uses some of the remaining heat in the flue gas to preheat the feed water before it enters the evaporator.

The illustrated model does not depict the flue gas cleaning and smoke stack and the conversion from steam power to electrical power is also left out. It is, however, assumed that when the plant is running at full load the electrical power produced amounts to 400 MW . Furthermore, the power plant is in this work augmented with two additional fuel systems: gas and oil.

The three different fuels have certain advantages and disadvantages, e.g., gas is easy to control, but is an expensive fuel. Some of the characteristics of the different fuels are:

Coal is advantageous when considering the price per stored energy; however, it is difficult to control as unmeasurable fluctuations in the coal flow are introduced when

the coal is ground to coal dust in the coal mill. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind the coal, which needs to be considered.

Gas is more expensive than coal and energy is not converted to steam as efficiently with gas as with coal due to the layout of the chosen boiler. However, gas arrives at the power plant under high pressure, which is lowered using a turbine generating electrical energy. Furthermore, gas is much easier to control as it is possible to measure the flow.

Oil is, with the current market prices, the most expensive of the three fuels and has to be pre-heated before entering the boiler. This process demands energy itself. Nevertheless, oil is considered in this work as it is possible to measure the oil flow into the boiler and this makes it easy to control. Furthermore, oil is present in most existing coal fired plants as oil is used to start up the plant.

In this work, it is assumed that the plant is controlled. Therefore, linear dynamics are sufficient to model it [13]. Indeed, it is shown in [13] that the change in the produced electricity caused by changing the fuel flow can be captured by third order dynamics. In the following, it is assumed that the fuel flow reference (in kg/s) of coal, gas, and oil is the input to the system, i.e., $\mathbf{u} = (u_c, u_g, u_o)$ is a vector of the coal, gas, and oil flow references respectively. The state vector consists of the actual flow of the different fuels and their first and second derivative, i.e., $\mathbf{z}_c = (z_1, z_2, z_3)$ is the coal flow into the boiler and its first and second derivative. Similarly, for the gas and oil systems $\mathbf{z}_g = (z_4, z_5, z_6)$ and $\mathbf{z}_o = (z_7, z_8, z_9)$. Therefore, the full state vector \mathbf{z} is given by $\mathbf{z} = (\mathbf{z}_c, \mathbf{z}_g, \mathbf{z}_o)$. Furthermore, the complete dynamics of the three different fuel systems considered in this work is given by

$$\begin{aligned}\dot{\mathbf{z}}(t) &= \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{x}(t) &= \mathbf{C}\mathbf{z}(t),\end{aligned}\tag{7.1}$$

where

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} \mathbf{A}_c & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{A}_g & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{A}_o \end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ h_{i1} & h_{i2} & h_{i3} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \mathbf{B}_c & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{B}_g & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{B}_o \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ h_{i0} \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} \mathbf{C}_1 & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{C}_1 & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{C}_1 \end{bmatrix}, \quad \mathbf{C}_1 = [1 \quad 0 \quad 0],\end{aligned}$$

and $h_{ij}, i \in \mathcal{I} = \{c, g, o\}$, are constants describing the dynamics of the three fuel systems, which are obtained from transfer functions of the form

$$H_i(s) = \frac{1}{(\tau_i s + 1)^3},$$

where $\tau_i, i \in \mathcal{I}$, is 90s, 60s, and 70s, respectively. See [12] for further comments on the above quantities.

The objective of this work is to derive a plan for optimal usage of the three fuels described above during 24 hours of operation. Optimal usage is defined as maximizing the profit in terms of two business objectives: efficiency and controllability. The models used in this work for business objectives are based on the following: Efficiency deals with a measure of efficiency of the conversion of a fuel into electricity² and controllability is a measure of the plant's capability to change the production level. In addition to maximizing the profit, the production level of the plant is to follow a time varying reference $y_r(t)$ (also called a production plan) as closely as possible.

The efficiency objective is modeled as

$$y_e(z) = \gamma^T \tilde{Q}z + \gamma^T b, \quad (7.2)$$

where

$$\begin{aligned} \tilde{Q} &= \text{diag}(e_x)C, \quad e_x = (10.77, 18.87, 15.77), \\ b &= (-1.76, 1.85, -0.37), \gamma = (1, 1, 1), \end{aligned}$$

with C as in (7.1). The value of the entries of e_x and b has been established using measurement data provided by DONG Energy³. The elements of e_x are conversion factors from mass flows to electrical-energy flows, and the entries of b are energy used or generated in preprocessing of the fuels.

The controllability objective is modeled as

$$y_c(z, t) = \vartheta(t)z + \zeta(t), \quad (7.3)$$

where

$$\begin{aligned} \vartheta(t) &= \begin{cases} 0 & y_r(t) \in S_1 = \{s \in \mathbb{R} | 0 \leq s \leq 200\} \\ \frac{\xi^T \tilde{Q}}{y_r(t)} & y_r(t) \in S_2 = \{s \in \mathbb{R} | 200 < s < 360\} \\ 0 & y_r(t) \in S_3 = \{s \in \mathbb{R} | 360 \leq s \leq 400\}, \end{cases} \\ \zeta(t) &= \begin{cases} 0.133 & y_r(t) \in S_1 \\ \frac{\xi^T b}{y_r(t)} & y_r(t) \in S_2 \\ 0.133 & y_r(t) \in S_3, \end{cases} \end{aligned}$$

with $\xi = (0.267, 0.534, 0.534)$ and S_1, S_2 , and S_3 denote different operating regions. The operating regions arise as maximum temperature gradients are imposed in the boiler due to wear and tear of the building materials. Therefore, the controllability measure also changes depending on the current production.

The value of the objectives have been established using price data available at Nord Pool and in collaboration with DONG Energy by using their heuristics. That is. current

²The model for efficiency in this work is often also referred to as production but as it depends on the efficiency of the power plant and fuel system this notation will be used in this work.

³DONG Energy is a power producer in Denmark

and historic prices of electricity, which available online has been used as price of the efficiency measure. The instantaneous profit flow is formulated as in [12], i.e.,

$$g_p(z, t) = \Theta(t)z + \tilde{\varphi}(t), \quad (7.4)$$

where

$$\begin{aligned} \Theta(t) &= p_{R1}(t)\gamma^T \tilde{Q} - p_C^T C + p_{R2}(t)\vartheta(t), \\ \tilde{\varphi}(t) &= p_{R1}(t)\gamma^T b + p_{R2}(t)\zeta(t), \end{aligned}$$

with p_C the price of the different fuels, and p_{R1} and p_{R2} are prices imposed on the two business objectives, efficiency and controllability respectively, as explained above quantities. The functions in (7.4) are in this work assumed sufficiently smooth (C^5 is enough as shown in Appendix 1). Further description and explanation of the above quantities can be found in [9], [10], [11], and [12].

A prognosis of the next days's electricity consumption is established by Energinet.dk⁴, which is responsible for the electrical grid in Denmark. The estimated electricity consumption in an area (e.g. Vest Denmark) is divided between the different electricity producers in accordance with the bids on Nord Pool; and thus, a production plan is generated for each producer. The production plan used in this work is an approximation of a production plan delivered by DONG Energy. These are depicted in Figure 7.2 (for more details on the production plan see [11]) The plant should follow the generated production plan

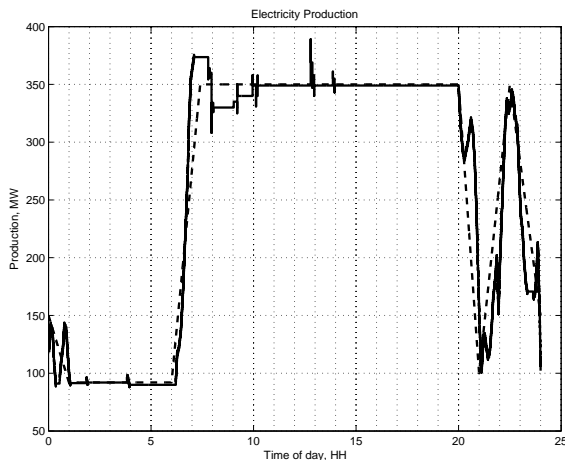


Figure 7.2: Production plan for June 29th, 2008. The data used to generate this plot has been delivered by DONG Energy (solid). The approximation used in this work consists of piece-wise affine function (dashed).

such that power balance can be upheld. The tracking requirement is included by adding

⁴Energinet.dk a Danish Transmission System Operator, TSO.

the quadratic tracking error,

$$\begin{aligned}
 t_e(\mathbf{z}, t) &= \|y_e(\mathbf{z}) - y_r(t)\|^2 \\
 &= \left\| \gamma^T \tilde{\mathbf{Q}}\mathbf{z} + \gamma^T \mathbf{b} - y_r(t) \right\|^2 \\
 &= \mathbf{z}^T \tilde{\mathbf{Q}}^T \gamma \gamma^T \tilde{\mathbf{Q}} \mathbf{z} - 2y_r(t) \gamma^T \tilde{\mathbf{Q}} \mathbf{z} + y_r(t)^2,
 \end{aligned} \tag{7.5}$$

as a penalty term in the objective function in the optimization. This approach has been taken as the computational complexity is lowered by including the requirement in the objective function and not as an additional constraint [14].

As a result, the optimization problem is expressed as

$$\begin{aligned}
 \max_{\mathbf{u} \in \mathcal{U}} \quad & \int_0^T f(\mathbf{z}, t) dt \\
 \text{subject to} \quad & \dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t),
 \end{aligned} \tag{7.6}$$

where the input space is given by

$$\mathcal{U} = \{\mathbf{u} \in \mathbb{R}_+^3 | \mathbf{e}_u^T \mathbf{u} \leq c_u, \mathbf{e}_u > 0\},$$

with $\mathbf{e}_u = \mathbf{e}_x$ and $c_u = 400 - \gamma^T \mathbf{b}$ as in [12], and f given by

$$\begin{aligned}
 f(\mathbf{z}, t) &= g_p(\mathbf{z}, t) - \beta_q t_e(\mathbf{z}, t) \\
 &= -\mathbf{z}^T \mathbf{Q} \mathbf{z} + 2\mathbf{q}(t)^T \mathbf{z} + \varphi(t),
 \end{aligned}$$

with

$$\begin{aligned}
 \mathbf{Q} &= \beta_q \tilde{\mathbf{Q}}^T \gamma \gamma^T \tilde{\mathbf{Q}} \\
 \mathbf{q}(t)^T &= \frac{1}{2} \boldsymbol{\Theta}(t) + \beta_q y_r(t) \gamma^T \tilde{\mathbf{Q}} \\
 \varphi(t) &= \tilde{\varphi}(t) - \beta_q y_r(t)^2,
 \end{aligned}$$

and β_q a positive weighting factor, which can be described as a reference penalty factor.

In the remaining sections, this paper deals with solving the optimization problem in (7.6).

3 Continuous Optimization

In this section, Pontryagin's maximum principle will be applied to the optimization problem described above.

However, first, remark that by replacing the (non-continuous) functions \mathbf{q} and φ by continuous approximations, such that f becomes continuous, Filippov's existence theorem [15, pp. 199] may be applied. Hence, in the continuous case, there exists an optimal solution to the optimization problem. Nonetheless, the question of existence for the discontinuous case will not be pursued here (see e.g. [15, p. 386] for a statement in this direction) since numerical solutions to the optimization problem are to be used, and since the above continuous approximations may be chosen with any given precision [16].

The Hamiltonian approach is now used, i.e., necessary conditions are deduced, by means of Pontryagin's maximum principle, to obtain candidates for optimal solutions.

The Hamiltonian for the optimization problem is

$$\begin{aligned} H(\mathbf{z}, \mathbf{u}, \boldsymbol{\lambda}, t) \\ = f(\mathbf{z}, t) + \boldsymbol{\lambda}^T (\mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u}), \end{aligned}$$

and thus, the adjoint equation is given by

$$\begin{aligned} \dot{\boldsymbol{\lambda}}(t) &= -\frac{\partial H(\mathbf{z}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t), t)}{\partial \mathbf{z}} \\ &= -\frac{\partial f(\mathbf{z}(t), t)}{\partial \mathbf{z}} - \mathbf{A}^T \boldsymbol{\lambda}(t) \\ &= 2\mathbf{Q}\mathbf{z}(t) - 2\mathbf{q}(t) - \mathbf{A}^T \boldsymbol{\lambda}(t), \end{aligned} \quad (7.7)$$

with the transversality condition $\boldsymbol{\lambda}(T) = \mathbf{0}$. Point-wise maximization of H then yields

$$\begin{aligned} \max_{\mathbf{u} \in \mathcal{U}} H(\mathbf{z}(t), \mathbf{u}, \boldsymbol{\lambda}(t), t) &= f(\mathbf{z}(t), t) + \boldsymbol{\lambda}(t)^T \mathbf{A}\mathbf{z}(t) \\ &\quad + \max_{\mathbf{u} \in \mathcal{U}} \underbrace{\boldsymbol{\lambda}(t)^T \mathbf{B}\mathbf{u}}_{\boldsymbol{\sigma}(t)^T}. \end{aligned}$$

Note that $\boldsymbol{\sigma}(t)$ is not known. However, by examining the sign of the coordinates of the vector $\boldsymbol{\sigma}(t)$, it is possible to determine the following properties of the optimal input, $\mathbf{u}^*(t)$,

$$\sigma_i(t) < 0 \Rightarrow u_i^*(t) = 0. \quad (7.8)$$

Now, let

$$\mathcal{U}(t) = \{\mathbf{u} \in \mathcal{U} \mid u_i = 0 \text{ if } \sigma_i(t) < 0\},$$

and let $\mathbf{E}(t)$ be the matrix which by projection removes the negative elements of $\boldsymbol{\sigma}(t)$, e.g., if $\sigma_1(t) < 0$ and $\sigma_2(t), \sigma_3(t) \geq 0$, we have

$$\mathbf{E}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note that the mapping $\mathbf{E}(t)$ is injective when restricted to $\mathcal{U}(t)$.

Let $\tilde{\boldsymbol{\sigma}}(t) = \mathbf{E}(t)\boldsymbol{\sigma}(t)$. Two cases now remain to be analyzed.

$$\begin{aligned} \text{Case 1: } \tilde{\boldsymbol{\sigma}}(t) &\geq \mathbf{0} \wedge \sum \tilde{\sigma}_i \neq 0. \\ \text{Case 2: } \tilde{\boldsymbol{\sigma}}(t) &= \mathbf{0}. \end{aligned} \quad (7.9)$$

In Case 1, the optimal control input $\mathbf{u}^*(t)$ is found from

$$\begin{aligned} \mathbf{u}^*(t) &= \arg \max_{\mathbf{u} \in \mathcal{U}(t)} \tilde{\boldsymbol{\sigma}}(t)^T \mathbf{E}(t)\mathbf{u} \\ \text{subject to } &\mathbf{e}_u^T \mathbf{u} = c_u, \end{aligned} \quad (7.10)$$

which, for each time t , searches through corners of a 2-simplex, 1-simplex, or 0-simplex in \mathbb{R}^3 .

Case 2 is an singular optimal control problem [17, 18]. The optimal control input in Case 2 is found from (26) in Appendix 1 as

$$\mathbf{C}_u(t)\mathbf{u}^*(t) = \mathbf{C}_z(t)\mathbf{z}(t) + \mathbf{C}_\tau(t), \quad \mathbf{u}^*(t) \in \mathcal{U}(t). \quad (7.11)$$

Here, the time dependence which was left out in the appendix is reintroduced as the entire time horizon is considered. Note that $\mathbf{C}_u(t)$ is not generally a square matrix. However, by introducing the following relation

$$\mathbf{u} = \mathbf{E}(t)^T \tilde{\mathbf{u}}, \quad (7.12)$$

and inserting it in (7.11),

$$\mathbf{C}_u(t)\mathbf{E}(t)^T \tilde{\mathbf{u}}(t) = \mathbf{C}_z(t)\mathbf{z}(t) + \mathbf{C}_\tau(t)$$

is obtained. Since $\mathbf{C}_u(t)\mathbf{E}(t)^T$ is square and non-singular, $\tilde{\mathbf{u}}(t)$ is given by

$$\begin{aligned} \tilde{\mathbf{u}}(t) = & (\mathbf{C}_u(t)\mathbf{E}(t)^T)^{-1} \mathbf{C}_z(t)\mathbf{z}(t) \\ & + (\mathbf{C}_u(t)\mathbf{E}(t)^T)^{-1} \mathbf{C}_\tau(t). \end{aligned}$$

Now, a $\mathbf{u}^*(t) \in \mathcal{U}(t)$ can again be constructed by using (7.12),

$$\begin{aligned} \mathbf{u}^*(t) = & \mathbf{E}(t)^T (\mathbf{C}_u(t)\mathbf{E}(t)^T)^{-1} \mathbf{C}_z(t)\mathbf{z}(t) \\ & + \mathbf{E}(t)^T (\mathbf{C}_u(t)\mathbf{E}(t)^T)^{-1} \mathbf{C}_\tau(t). \end{aligned} \quad (7.13)$$

In conclusion, in Case 1, the optimal control input is an open loop controller; and in Case 2, a combination of a piece-wise constant state feedback and a time varying feedforward. The combination of Case 1 and Case 2 will in the following be denoted “feedback/feedforward”.

Now, a strategy for finding the optimal control input is devised; however, the switching function $\sigma(t)$, which is required in the control law, is unknown and thus the time instances of switching between the two cases (and different $\mathbf{E}(t)$) can not be determined.

The next two sections of this paper will present a solution to this problem. It will be based on an approximated solution to the optimization problem. The approximated solution will then be used to solve the adjoint equation and thus obtain an approximated solution for $\sigma(t)$. The approximated solution will be found using a discrete time formulation of the optimization problem. This procedure is explained in the next section.

4 Discrete Optimization

The optimization problem in (7.6) has been addressed in [12] where the tracking of the reference was formulated as a constraint. As a consequence, it has not been modeled in the objective function. This is done in this work. In this section, we will apply the procedure in [12] to the quadratic objective function, i.e., we will formulate the optimization problem in discrete time. In Appendix 2, lifting [19] is applied to the objective function in (7.6) to obtain a discrete-time expression. However, some assumptions are imposed. In particular, it is assumed that $\mathbf{q}(t)$ and $\varphi(t)$ can be approximated by piecewise constant functions, and that the control \mathbf{u} is piecewise constant.

The discrete time objective function can be formulated as

$$\sum_{k=0}^{N-1} \left(\begin{bmatrix} \mathbf{z}_k^T & \mathbf{u}_k^T \end{bmatrix} \mathbf{N} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} + \mathbf{M}_z \mathbf{z}_k + \mathbf{M}_u \mathbf{u}_k + h\varphi_k \right),$$

where

$$\mathbf{N} = - \begin{bmatrix} \mathbf{N}_{zz} & \mathbf{N}_{zu} \\ \mathbf{N}_{uz} & \mathbf{N}_{uu} \end{bmatrix},$$

with the matrices \mathbf{N}_{zz} , \mathbf{N}_{zu} , \mathbf{N}_{uz} , \mathbf{N}_{uu} , \mathbf{M}_z , and \mathbf{M}_u as given by (31) and (37) in Appendix 2. The optimization problem in (7.6) can be rewritten by introducing the following notation

$$\tilde{\Phi} = \begin{bmatrix} \mathbf{I} \\ \Phi \\ \Phi^2 \\ \vdots \\ \Phi^{N-2} \end{bmatrix}, \quad \tilde{\varphi} = \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_{N-1} \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix},$$

$$\tilde{\Gamma} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \Gamma & \mathbf{0} & \cdots & \mathbf{0} \\ \Phi\Gamma & \Gamma & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \Phi^{N-3}\Gamma & \Phi^{N-4}\Gamma & \cdots & \Gamma \end{bmatrix},$$

and the matrices $\Phi = e^{\mathbf{A}h}$ and $\Gamma = \int_0^h e^{\mathbf{A}s} \mathbf{B} ds$ are the discrete time equivalences of the system matrices given in (7.1). Furthermore, φ_k is given by (28) in Appendix 2.

Using the above discrete time system and considering \mathbf{z}_k as a function of \mathbf{z}_0 and \mathbf{u}_k , it is possible to formulate the following optimization problem, which is the discrete time equivalent of (7.6)

$$\max_{\mathbf{v} \in \mathcal{V}} \mathbf{v}^T \mathbf{W} \mathbf{v} + \mathbf{L} \mathbf{v} + g, \quad (7.14)$$

where

$$\begin{aligned}
 \mathbf{v} &= \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{N-2} \end{bmatrix}, \\
 \mathcal{V} &= \{\mathbf{v} \in \mathbb{R}^{m(N-1)} | \mathbf{v}_i \in \mathcal{U}, i \in \{0, 1, 2, \dots, N-2\}\}, \\
 \mathbf{W} &= -\tilde{\mathbf{\Gamma}}^T (\mathbf{I} \otimes \mathbf{N}_{zz}) \tilde{\mathbf{\Gamma}} - \tilde{\mathbf{\Gamma}}^T (\mathbf{I} \otimes \mathbf{N}_{zu}) \\
 &\quad - (\mathbf{I} \otimes \mathbf{N}_{uz}) \tilde{\mathbf{\Gamma}} - \mathbf{I} \otimes \mathbf{N}_{uu}, \\
 \mathbf{L} &= \mathbf{1}^T \otimes \mathbf{M}_u + (\mathbf{1}^T \otimes \mathbf{M}_z) \tilde{\mathbf{\Gamma}} \\
 &\quad - \mathbf{z}_0^T \tilde{\mathbf{\Phi}}^T (\mathbf{I} \otimes \mathbf{N}_{zu} + \mathbf{I} \otimes \mathbf{N}_{uz}^T) \\
 &\quad - \mathbf{z}_0^T \tilde{\mathbf{\Phi}}^T (\mathbf{I} \otimes \mathbf{N}_{zz} + \mathbf{I} \otimes \mathbf{N}_{zz}^T) \tilde{\mathbf{\Gamma}}, \\
 g &= (\mathbf{1}^T \otimes \mathbf{M}_z) \tilde{\mathbf{\Phi}} \mathbf{z}_0 + h \mathbf{1}^T \tilde{\varphi} - \mathbf{z}_0^T \tilde{\mathbf{\Phi}}^T (\mathbf{I} \otimes \mathbf{N}_{zz}) \tilde{\mathbf{\Phi}} \mathbf{z}_0,
 \end{aligned}$$

with \otimes the Kronecker product.

The optimization in (7.14) has been implemented using YALMIP with the following constants used for the parameters in (31) and (37)

$$h = 192s, \quad \beta_q = 0.05, \quad T = 86400s, \quad N = 450,$$

and solved using the quadratic solver BPPMD. Remark that the sampling time of 192 s yielding 450 discrete points in time is close to the limit of the capabilities of the computer used for the optimization.

Figure 7.3-top depicts the profit and the value of the objective function. Remark that the word “profit” is used for the real profit of the company, i.e., the objective function value without the quadratic tracking penalty, t_e , and the expression “value of the objective function” is used when the quadratic tracking penalty is included. The profit computed in this work is in the same order of magnitude as the results obtained in [12] where the tracking requirement was implemented as a constraint instead of included in the objective function as in this work.

The usage of fuels are also comparable to [12] except that around 7:00 when the gas system is not used, as seen in Figure 7.3-bottom.

5 Optimal Feedback

In this section, the idea of a continuous feedback/feedforward from Section 3 is revisited. The reason for this is the usual robust behavior of a feedback system with regards to noise compared with pure feedforward control (this is further discussed in Section 6).

Let us begin by remarking that an essential part of the optimization problem is to follow the predefined reference. In particular, there should be no market situation such that a large deviation from reference is beneficial. This, of course, depends on the value of β which in this work has been chosen such that the above is satisfied.

As stated at the end of Section 3, the reason for not using the feedback/feedforward solution is that $\sigma(t)$ is unknown. To compensate for this, we introduce an algorithm

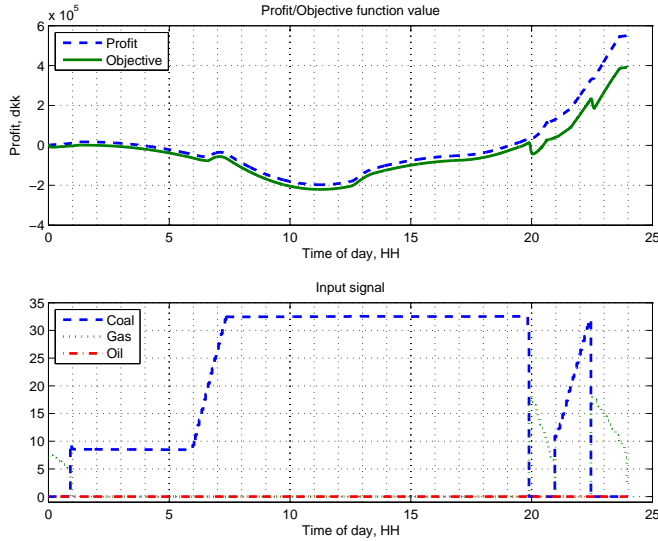


Figure 7.3: The result of the discrete optimization. Top: The profit and value of objective function during 24 hours. Bottom: The control signal for the different fuel systems.

whose purpose is to approximate $\sigma(t)$. In short, we need to approximate $\lambda(t)$ which depends on $z(t)$. As an input for the algorithm we use the discretized solution $z(t) = (z_0, \dots, z_{N-2})$ given by $\tilde{\Phi}z_o + \tilde{\Gamma}v$ as described in the previous section. Note that $z(t)$ can be viewed as a small perturbation of the optimal solution $z^*(t)$. As a result, the algorithm is, as follows:

1. Use $z(t)$ to obtain $\lambda(t)$ in (7.7) with transversality condition $\lambda(T) = 0$. In simulations, this step is preformed using Matlab's ode45.
2. Use $\lambda(t)$ to compute $\sigma(t) = B^T \lambda(t)$ and the projection $E(t)$ as described in Section 3.
3. For each time t determine, by evaluating $\sigma(t)$, whether case 1 or 2 in (7.9) holds:
 - I. If case 1 use (7.10) to compute $u^*(t)$.
 - II. If case 2 use $\lambda(t)$ and $E(t)$ to compute $C_i(t)$, $i = z, u, t$ given by (27). Then compute $u^*(t)$ using (7.13). We remark that due to numerical imprecision, in simulations, we have placed a band around 0 of width 1 in which all elements are set to zero.
4. Use $u^*(t)$ to obtain $z(t)$ in (7.1). In simulations, this step is preformed using Matlab's ode45.
5. Return to 1.

The switch function, $\sigma(t) = B^T \lambda(t)$ is depicted in Figure 7.4, where the solution to the adjoint equation $\lambda(t)$ is computed using the discrete state trajectory $(z(t_k))$, $t_k \in$

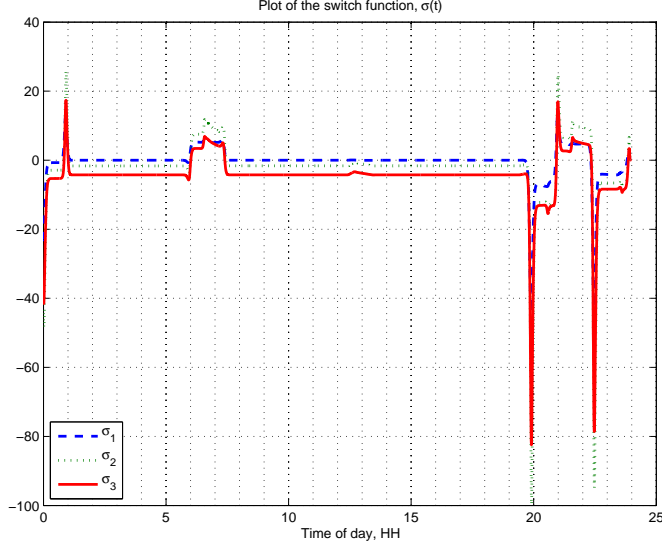


Figure 7.4: Graph of the switch function obtained by using the state trajectory from the discrete optimization.

$\{0, h, 2h, \dots, (N-1)h\}$, i.e., the first iteration of the algorithm described above. After 300 iterations, the above algorithm shows no sign of convergence. This is illustrated in Figure 7.5, where the graph of $\sigma(t)$ is depicted for iteration 295-300. The solutions switch between two different profiles, this can be explained as follows: First, note that the adjoint equation is solved backwards and hence an equivalent problem can be stated as

$$\dot{\Lambda} = A^T \Lambda + c, \quad (7.15)$$

with

$$\begin{aligned} c &= -2Qz(t) + 2q(t) \\ &= -2\tilde{Q}^T \gamma(y_e(z(t)) + \beta y_r(t)) + \Theta(t). \end{aligned} \quad (7.16)$$

Further, note that A^T is stable and that $y_r(t)$ and $\Theta(t)$ are not affected by the iterations. Now, if σ is positive in iteration k as a consequence of c being positive in iteration $k-1$, then in iteration k this results in maximum efficiency as follows from (7.10). This results in a negative c , and thus, the solution of (7.15) will also become negative. In conclusion, c oscillates. This implies the divergence of the algorithm.

Moreover, the reason for the oscillating behavior of the algorithm described above is illustrated in Figure 7.4. More precisely, the initial estimate of $\sigma(t)$ is less than zero in $[0, 0:15]$. According to (7.8), $u(t) = \mathbf{0}$, $t \in [0, 0:15]$. As a result, the efficiency output y_e deviates from the reference y_r causing the increase of the error calculated by (7.5). This behavior is also present in the intervals $[19:45, 21]$ and $[22:30, 24]$ and in the intervals $[0:45, 1:05]$ $[6, 7:30]$ and $[21, 22:30]$. Due to the choice of β , as discussion

above, this behavior cannot result in an optimal solution. Therefore, subsequent iterations will not improve the estimate of the parameters, and in particular not $\sigma(t)$ as seen in Figure 7.5. Remark that it is well known that numerical problems may appear in problems with singular arcs. In conclusion the above method has to be modified in order to obtain reference tracking as discussed above.

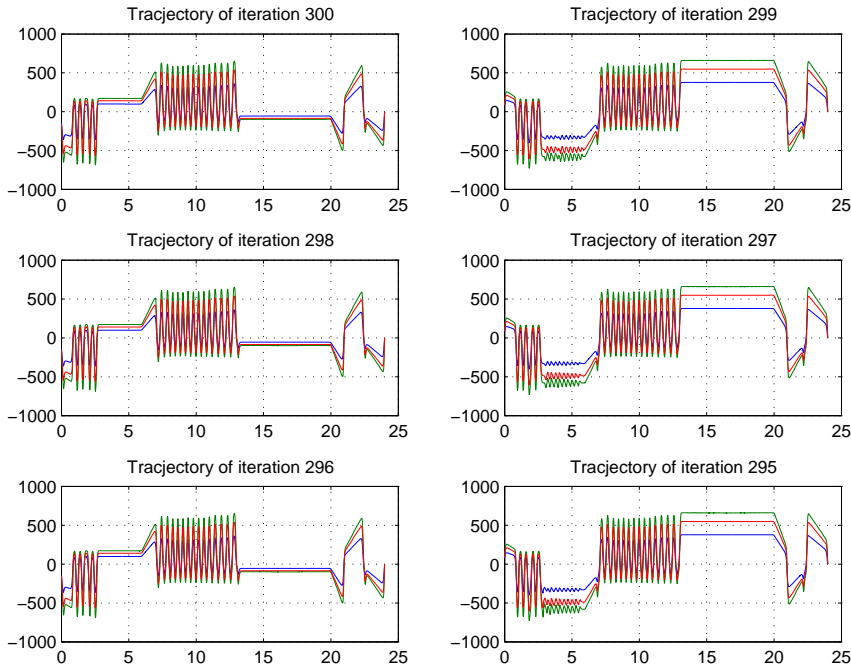


Figure 7.5: Graph of the switch function calculated in iterations 295-300.

Some heuristics are, therefore, introduced to avoid the reference deviations (and non-optimal behavior). From the discrete optimization, it is observed that only one fuel is used at the time, which has also been suggested by earlier work [12, 11]. As a consequence, $\sigma_{i,j}(t) < 0$ at any given time, where the notation $x_{i,j}$ means coordinates i and j of the vector x . Therefore, the introduced heuristics is to use only the one element of $\mathbf{u}(t)$ corresponding to the largest element of $\sigma(t)$. To recapitulate, item 3. in the above algorithm is replaced with

- 3'. Determine the largest element of $\sigma(t)$, say $\sigma_l(t)$, and let $E(t)$ be the 1×3 matrix with 1 at place l and zero's elsewhere, and then, use (7.13) to compute $u^*(t)$.

Using this concept it is possible to obtain an control input as described in Section 3 which yields better behavior. When applied, this input strategy will provide another state trajectory and thus a different trajectory of $\sigma(t)$. For the given case study, this procedure stabilizes⁵, which might not be true in general. Remark that in order to conduct a rigorous mathematical discuss about the convergence properties, a mathematical model of the

⁵The procedure has been executed 30 times and after step four it stabilizes.

above algorithm has to be devised. Such a model can only be an approximation of the above algorithm due to the introduced heuristics when defining $E(t)$. This is further complicated by the discrete behavior of $\sigma(t)$ and $E(t)$. However, by the heuristic construction of $E(t)$, reference tracking is maintained. This implies that (7.16) becomes

$$\mathbf{c} = -2\tilde{\mathbf{Q}}^T \gamma (\epsilon + (1 - \beta)y_r(t)) + \Theta(t)$$

for small ϵ , i.e. \mathbf{c} is not affected by the iterations due to reference tracking. As a result, only small perturbation of the stable linear system (7.15) is introduced in each iteration. This results in convergence of the proposed algorithm.

In this paper, the first four iterations of this procedure has been applied, where the state trajectory from the discrete optimization is used for the initial iteration. Figure 7.6 depicts the convergence of the switch function from the different iterations, where σ_{11} denotes coordinate one for iteration one, σ_{12} coordinate one from iteration two, etc. As

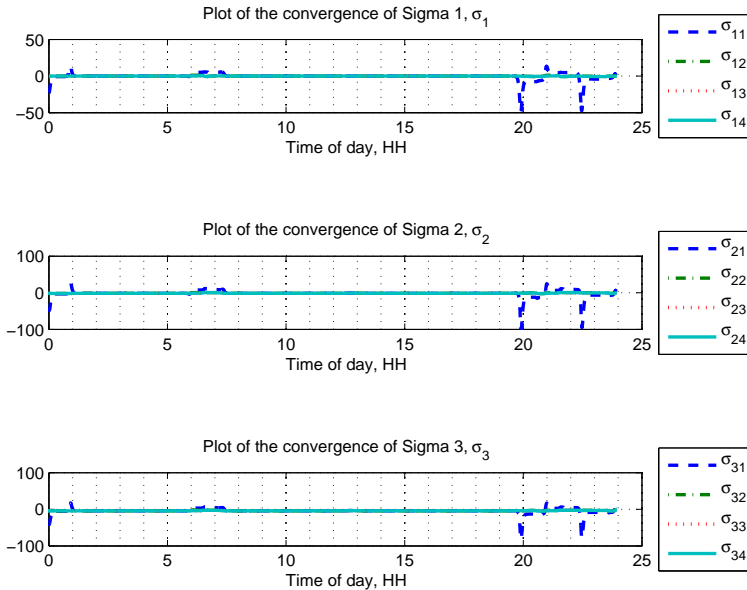


Figure 7.6: Convergence of the coordinates of the switch function - four iterations are depicted. Note that iteration two, three, and four are almost on top of each other.

seen in the figure, Coordinate three always converges to a value less than zero. Thus, the oil system should not be used at all and could be omitted when the plant is instrumented. Coordinate two is less than zero for large periods during the day; hence, gas should not be used during these periods. Coordinate one, on the other hand, is zero most of the time.

The input strategy for each of the four iterations has been applied to the model of the plant. Figure 7.7 depicts the graphs of the resulting objective function values for the four iterations of the adjoint equation along with the objective function value from the discrete optimizations. The legends refer to the value of the discrete objective function and continuous objective function from iteration 1, 2, 3, and 4. As seen in the figure,

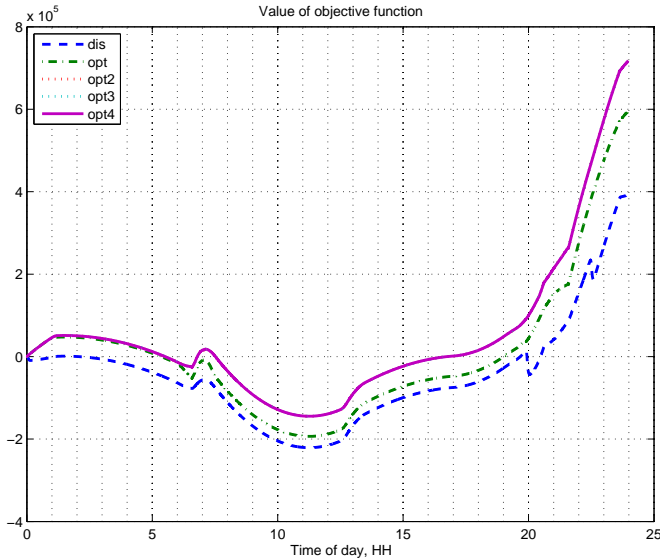


Figure 7.7: The value of the objective function when using the input from the discrete optimization and the four iterations of the switch function. Note that the graph of iteration two, three, and four are almost on top of each other.

the value of the objective function increases substantially from the discrete to the first iteration and from the first iteration to the second iteration. However, iteration three and four do not change the value of the objective function significantly.

The final input strategy is evaluated in Figure 7.8, where the top figure depicts the profit and objective function value, and the bottom figure depicts optimal input. As seen in the top figure, the profit and objective function values are very close to each other. Thus, the production tracks the reference closely when the optimal feedback/feedforward is used. The final profit is approximately 168000 $[dkk]$ or 30% larger than what was obtained using the input from the discrete optimization. Furthermore, the optimal input is different from the input obtained in the discrete optimization. This is specially seen in the usage of gas in the period 6:30-7:00 as depicted in bottom figure. The spikes in the input signal is due to the discontinuous switches in (7.3). However, these spikes do not affect the output as they are of very short time span. Furthermore, notice that gas is used briefly in the period 20:20-20:40. Obviously, it might not be feasible in practice to switch between system within very short time intervals. This could be circumvented by adding a cost of switching a fuel system on, but this is regarded as outside the scope of this work. Yet, 20 minutes is believed to be sufficient time considering a sampling time of 10 seconds.

6 Noise and System Uncertainty

The performance of the two solutions presented above will in this section be discussed with respect to noise and system uncertainty. As noted previously, a system with feedback

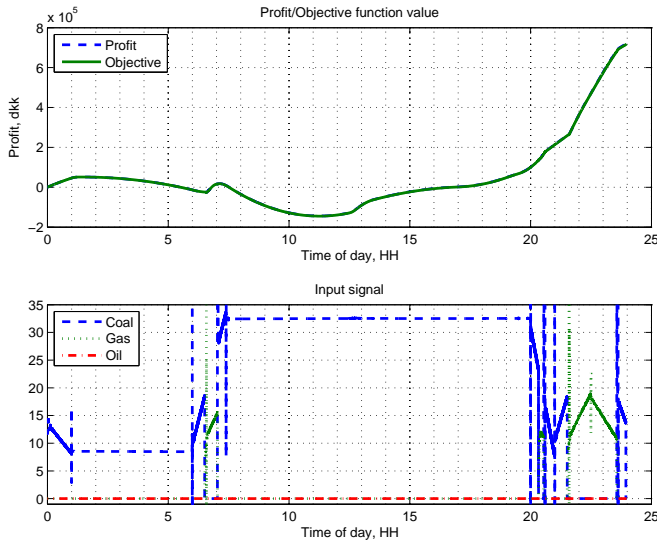


Figure 7.8: Result of using the optimal input from the fourth iteration. Top: The profit and value of objective function during 24 hours. Bottom: The control signal for the different fuel systems. The spikes in the control signal arise from the switching between the sets S_1 , S_2 , and S_3 .

is typically more robust towards system noise than with feedforward control. To evaluate this supposition, input noise will be considered. The noise is in this work assumed to be Gaussian white noise with a standard deviation of 5% of the maximum input signal.

The value of the objective function during 24 hours with noise is depicted in Figure 7.9. As seen in the figure, the value is lower than without noise for both the discrete case and the optimal feedback/feedforward. However, the value of the objective function when using optimal feedback/feedforward is substantially larger than when using the discrete input. The reason for this is that the optimal feedback/feedforward tracks the reference better.

The tracking errors for the two input strategies are depicted in Figure 7.10. As seen in the figure, the error using the optimal feedback/feedforward is smaller than the error resulting from the discrete input. The mean tracking error is 3.40 [MW] when using the optimal feedback/feedforward and 14.73 [MW] when using the input from the discrete optimization. Furthermore, the standard deviation of the signal is also smaller using the feedback/feedforward solution as the values are 2.54 [MW] and 6.54 [MW] for feedback/feedforward and discrete, respectively.

The optimal feedback/feedforward is also superior in presence of noise wrt. the profit of the company. However, the difference is not as significant as when the tracking term is included. This can be observed in Figure 7.11 where the economical profit for the two solutions are depicted. The difference is about 161000 [dkk], and thus, the gain of using the optimal feedback/feedforward is approximately 45%.

When system uncertainties are introduced, both methods deteriorate equally much; in the sense that, some stationary tracking error arises, and thus, the value of the objective

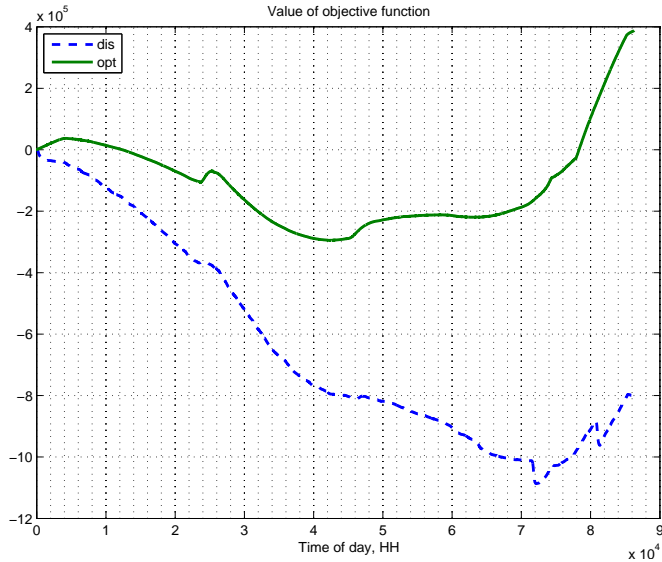


Figure 7.9: The value of the objective function using the input obtained from the discrete and continuous optimization when input noise is present.

function is lowered in both cases.

7 Discussion

In this work, Pontryagin's maximum principle has been applied to a problem dealing with profit maximization of a power plant. An optimal input strategy consisting of a combined feedback and feedforward has been developed such that profit is maximized over 24 hours of operation. The developed strategy is based on properties of the optimal control input and an initial solution of the adjoint equations obtained from discrete optimization. The two solutions, discrete input strategy and continuous feedback/feedforward, are evaluated both with and without input noise. As a result, the optimal feedback/feedforward yields a greater profit in both cases. In the presence of input noise, the feedback/feedforward solution yields a profit 45% larger than what is possible by using the discrete input strategy.

Future work in line with this paper would include improving the initial estimate of the switch function. In particular, a method using pseudo-spectral techniques to obtain a solution to the adjoint equation is proposed [20]. Using this method, it might be possible to obtain a sufficiently accurate estimate of the switch function $\sigma(t)$ within less iterations of the algorithm suggested in this work. This could decrease the computational complexity and solve time making the proposed method interesting for online implementation as a receding horizon.

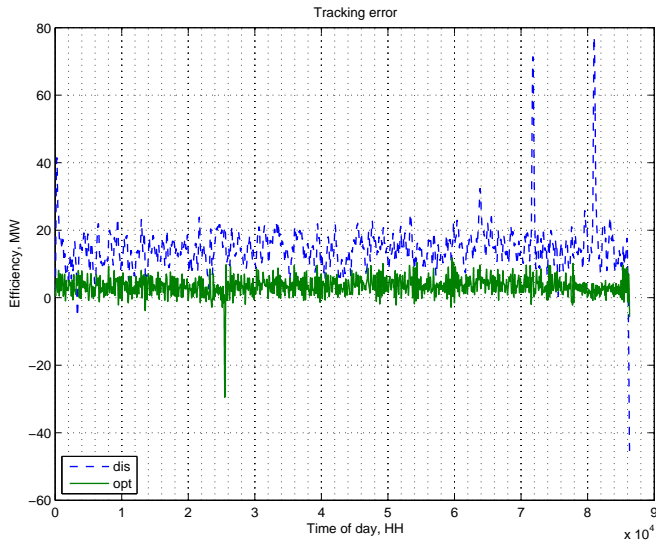


Figure 7.10: The tracking error resulting from using the input obtained from the discrete and continuous optimization.

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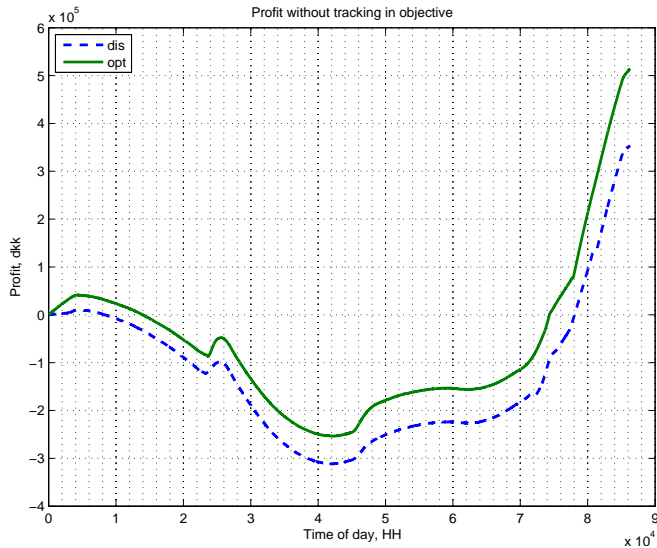


Figure 7.11: The profit obtained during 24 hours when input noise is present.

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1

In this appendix, the optimal control input is found by examining the singular solution [17, 18], i.e., the equality

$$\tilde{\sigma}(t) = \mathbf{E}(t)\mathbf{B}^T\boldsymbol{\lambda}(t) \equiv \mathbf{0} \quad (17)$$

is considered point-wise on a nondegenerate time interval. The matrix $\mathbf{E}(t)$ is constant in this time interval and will in the following be denoted by \mathbf{E} to avoid confusion when taking the time derivative.

Some notation is introduced to simplify the equations throughout this appendix

$$\mathbf{R}(r, s, l) = 2\mathbf{E}\mathbf{B}^T \mathbf{A}^{rT} \mathbf{Q}^s \mathbf{A}^l \quad (18)$$

$$\mathbf{D}(i, t) = \sum_{n=0}^i (-1)^n \mathbf{R}(n, 0, 0) \mathbf{q}^{(i-n)}(t), \quad (19)$$

where $\mathbf{A}^{nT} = \prod_{j=1}^n \mathbf{A}^T$, with $\mathbf{A}^{0T} = \mathbf{I}$ and $\mathbf{q}^{(j)}(t)$ is the j th time derivative with $\mathbf{q}^{(0)}(t) = \mathbf{q}(t)$.

Differentiation of (17) yields

$$\dot{\tilde{\sigma}}(t) = \mathbf{E}\mathbf{B}^T \dot{\boldsymbol{\lambda}}(t) = \mathbf{0}$$

Inserting the adjoint equation, (7.7), yields

$$\begin{aligned} \dot{\tilde{\sigma}}(t) &= \mathbf{E}\mathbf{B}^T \left(2\mathbf{Q}\mathbf{z}(t) - 2\mathbf{q}(t) - \mathbf{A}^T\boldsymbol{\lambda}(t) \right) = \mathbf{0} \Leftrightarrow \\ \mathbf{D}(0, t) &= \underbrace{\mathbf{R}(0, 1, 0)}_{\mathbf{0}} \mathbf{z}(t) - \frac{\mathbf{R}(1, 0, 0)\boldsymbol{\lambda}(t)}{2}, \end{aligned} \quad (20)$$

where $\mathbf{R}(0, 1, 0) = \mathbf{0}$ follows from the structure of \mathbf{Q} , \mathbf{B} and their sparsity, i.e., the relative degrees of the individual fuel systems are 3; and thus, the first and second time derivative will yield zero. Differentiating once more and inserting the adjoint equation yields the following equalities

$$\begin{aligned}\dot{D}(0, t) &= -\frac{\mathbf{R}(1, 0, 0)\dot{\lambda}(t)}{2} \\ \dot{D}(0, t) &= -\mathbf{R}(1, 0, 0)\mathbf{Q}z(t) + \mathbf{R}(1, 0, 0)q(t) \\ &\quad + \frac{\mathbf{R}(1, 0, 0)\mathbf{A}^T\lambda(t)}{2} \\ D(1, t) &= -\underbrace{\mathbf{R}(1, 1, 0)}_{\mathbf{0}}z(t) + \frac{\mathbf{R}(2, 0, 0)\lambda(t)}{2},\end{aligned}\tag{21}$$

where $\mathbf{R}(1, 1, 0) = \mathbf{0}$ follows from the structure of \mathbf{Q} , \mathbf{B} , and \mathbf{A} as with $\mathbf{R}(0, 1, 0) = \mathbf{0}$ above. Now, it is possible to determine $\lambda(t)$ using (17), (20), and (21). However, the objective is to find the optimal control input, therefore, (21) is differentiated once more which yields

$$\begin{aligned}\dot{D}(1, t) &= \frac{\mathbf{R}(2, 0, 0)\dot{\lambda}(t)}{2} \\ \dot{D}(1, t) &= \mathbf{R}(2, 0, 0)\mathbf{Q}z(t) - \mathbf{R}(2, 0, 0)q(t) \\ &\quad - \frac{\mathbf{R}(2, 0, 0)\mathbf{A}^T\lambda(t)}{2} \\ D(2, t) &= \mathbf{R}(2, 1, 0)z(t) - \frac{\mathbf{R}(3, 0, 0)\lambda(t)}{2}\end{aligned}\tag{22}$$

when the adjoint equations is inserted. As $\mathbf{R}(2, 1, 0) \neq \mathbf{0}$, (22) is differentiated again and the adjoint and system equations, (7.7) and (7.1), are inserted which yields

$$\begin{aligned}\dot{D}(2, t) &= \mathbf{R}(2, 1, 0)\dot{z}(t) - \frac{\mathbf{R}(3, 0, 0)\dot{\lambda}(t)}{2} \\ \dot{D}(2, t) &= \mathbf{R}(2, 1, 0)(\mathbf{A}z(t) + \mathbf{B}u(t)) \\ &\quad - \frac{\mathbf{R}(3, 0, 0)(2\mathbf{Q}z(t) - 2q(t) - \mathbf{A}^T\lambda(t))}{2} \\ \dot{D}(2, t) &= \mathbf{R}(2, 1, 0)(\mathbf{A}z(t) + \mathbf{B}u(t)) \\ &\quad - \mathbf{R}(3, 0, 0)\mathbf{Q}z(t) + \mathbf{R}(3, 0, 0)q(t) \\ &\quad + \frac{\mathbf{R}(3, 0, 0)\mathbf{A}^T\lambda(t)}{2} \\ D(3, t) &= (\mathbf{R}(2, 1, 1) - \mathbf{R}(3, 1, 0))z(t) \\ &\quad + \underbrace{\mathbf{R}(2, 1, 0)\mathbf{B}}_{\mathbf{0}}u(t) + \frac{\mathbf{R}(4, 0, 0)\lambda(t)}{2},\end{aligned}\tag{23}$$

where $\mathbf{R}(2, 1, 0)\mathbf{B} = \mathbf{0}$ is attributed the relative degree equals 3. Equation (23) is differ-

entiated again which yields

$$\begin{aligned}
 \dot{D}(3, t) &= (\mathbf{R}(2, 1, 1) - \mathbf{R}(3, 1, 0)) \dot{\mathbf{z}}(t) \\
 &\quad + \frac{\mathbf{R}(4, 0, 0) \dot{\lambda}(t)}{2} \\
 \dot{D}(3, t) &= (\mathbf{R}(2, 1, 1) - \mathbf{R}(3, 1, 0)) (\mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t)) \\
 &\quad + \frac{\mathbf{R}(4, 0, 0) (2\mathbf{Q}\mathbf{z}(t) - 2\mathbf{q}(t) - \mathbf{A}^T \lambda(t))}{2} \\
 \dot{D}(3, t) &= (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1)) \mathbf{z}(t) \\
 &\quad + \underbrace{(\mathbf{R}(2, 1, 1) - \mathbf{R}(3, 1, 0)) \mathbf{B}}_{\mathbf{0}} \mathbf{u}(t) \\
 &\quad + \mathbf{R}(4, 1, 0) \mathbf{z}(t) - \mathbf{R}(4, 0, 0) \mathbf{q}(t) \\
 &\quad - \frac{\mathbf{R}(5, 0, 0) \lambda(t)}{2} \\
 D(4, t) &= (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{z}(t) \\
 &\quad - \frac{\mathbf{R}(5, 0, 0) \lambda(t)}{2}, \tag{24}
 \end{aligned}$$

and $(\mathbf{R}(2, 1, 1) - \mathbf{R}(3, 1, 0)) \mathbf{B} = \mathbf{0}$ due to the relative degree of 3. Using (22), (23), and (24) it is possible to calculate $\mathbf{z}(t)$. To obtain an expression for $\mathbf{u}(t)$, equation (24) is differentiated and adjoint and system equations are inserted

$$\begin{aligned}
 \dot{D}(4, t) &= (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \dot{\mathbf{z}}(t) \\
 &\quad - \frac{\mathbf{R}(5, 0, 0) \dot{\lambda}(t)}{2} \\
 \dot{D}(4, t) &= (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{A}\mathbf{z}(t) \\
 &\quad + (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{B}\mathbf{u}(t) \\
 &\quad - \frac{\mathbf{R}(5, 0, 0) (2\mathbf{Q}\mathbf{z}(t) - 2\mathbf{q}(t) - \mathbf{A}^T \lambda(t))}{2} \\
 \dot{D}(4, t) &= (\mathbf{R}(2, 1, 3) - \mathbf{R}(3, 1, 2) + \mathbf{R}(4, 1, 1)) \mathbf{z}(t) \\
 &\quad + (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{B}\mathbf{u}(t) \\
 &\quad - \mathbf{R}(5, 0, 0) \mathbf{Q}\mathbf{z}(t) + \mathbf{R}(5, 0, 0) \mathbf{q}(t) \\
 &\quad + \frac{\mathbf{R}(5, 0, 0) \mathbf{A}^T \lambda(t)}{2} \\
 D(5, t) &= (\mathbf{R}(2, 1, 3) - \mathbf{R}(3, 1, 2) \\
 &\quad + \mathbf{R}(4, 1, 1) - \mathbf{R}(5, 1, 0)) \mathbf{z}(t) \\
 &\quad + (\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{B}\mathbf{u}(t) \\
 &\quad + \frac{\mathbf{R}(6, 0, 0) \lambda(t)}{2} \tag{25}
 \end{aligned}$$

Using (25), it is possible to obtain an expression for $\mathbf{u}(t)$

$$\mathbf{C}_u \mathbf{u}(t) = \mathbf{C}_z \mathbf{z}(t) + \mathbf{C}_\tau(t), \quad \mathbf{u}(t) \in \mathcal{U}(t) \tag{26}$$

where

$$\begin{aligned} C_u &= -(\mathbf{R}(2, 1, 2) - \mathbf{R}(3, 1, 1) + \mathbf{R}(4, 1, 0)) \mathbf{B} \\ C_z &= \mathbf{R}(2, 1, 3) - \mathbf{R}(3, 1, 2) + \mathbf{R}(4, 1, 1) - \mathbf{R}(5, 1, 0) \\ C_\tau(t) &= \frac{\mathbf{R}(6, 0, 0)\lambda(t)}{2} - \mathbf{D}(5, t). \end{aligned} \quad (27)$$

2

In this appendix, we will derive the lifting of the cost function, which is used in the discrete optimization. The term

$$P(T) = \underbrace{\int_0^T -\mathbf{z}(t)^T \mathbf{Q} \mathbf{z}(t) dt}_{P_2(T)} + \underbrace{\int_0^T 2\mathbf{q}(t)^T \mathbf{z}(t) + \varphi(t) dt}_{P_1(T)}$$

is divided into two part during the discretization, i.e., the quadratic and affine terms are treated separately.

It is assumed that $\mathbf{q}(t)$ and $\varphi(t)$ can be approximated by piecewise constant functions for each time step, i.e.,

$$\begin{aligned} \mathbf{q}(t) &= \mathbf{q}_k, \quad kh < t < (k+1)h, \\ \varphi(t) &= \varphi_k, \quad kh < t < (k+1)h, \end{aligned} \quad (28)$$

where h is the sampling time. Furthermore, the control is assumed piecewise constant as customary when digital to analogue conversion is performed using sample-and-hold circuits.

Using a fact from [21], the continuous time state $\mathbf{z}(t)$ in the dynamic system in (7.1) with constant input \mathbf{u}_0 can be described by

$$\begin{aligned} \mathbf{z}(t) &= e^{\mathbf{A}t} \mathbf{z}_0 + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B} \mathbf{u}_0(s) ds \\ &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \exp \left\{ \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} t \right\} \begin{bmatrix} \mathbf{z}_0 \\ \mathbf{u}_0 \end{bmatrix}, \end{aligned} \quad (29)$$

where \mathbf{I} is an identity matrix with appropriate dimension and \mathbf{z}_0 is the initial state. Using (29), it is possible to derive the following formula

$$\begin{aligned} \int_0^h e^{\mathbf{A}t} dt &= e^{\mathbf{A}h} \int_0^h e^{-\mathbf{A}(h-t)} dt \\ &= e^{\mathbf{A}h} \left(e^{-\mathbf{A}h} \cdot 0 + \int_0^h e^{-\mathbf{A}(h-t)} \mathbf{I} dt \right) \\ &= e^{\mathbf{A}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \exp \left\{ \begin{bmatrix} -\mathbf{A} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} h \right\} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \end{aligned} \quad (30)$$

The affine term is lifted by first using (29) and then (30)

$$\begin{aligned}
P_1(T) &= \int_0^T \left(2\mathbf{q}(t)^T \mathbf{z}(t) + \varphi(t) \right) dt \\
&= \sum_{k=0}^{N-1} 2\mathbf{q}(t)^T \int_0^h \left(e^{\mathbf{A}t} \mathbf{z}_k + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B} ds \mathbf{u}_k \right) dt \\
&\quad + h \sum_{k=0}^{N-1} \varphi_k \\
&= \sum_{k=0}^{N-1} 2\mathbf{q}(t)^T \int_0^h \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}t} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} dt + h \sum_{k=0}^{N-1} \varphi_k \\
&= \sum_{k=0}^{N-1} 2\mathbf{q}(t)^T e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} + h \sum_{k=0}^{N-1} \varphi_k \\
&= \sum_{k=0}^{N-1} (\mathbf{M}_z \mathbf{z}_k + \mathbf{M}_u \mathbf{u}_k + h\varphi_k), \tag{31}
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{M}_z &= 2\mathbf{q}(t)^T e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{M}_u &= 2\mathbf{q}(t)^T e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}
\end{aligned}$$

with

$$\hat{\mathbf{A}} = \begin{bmatrix} -\tilde{\mathbf{A}} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

and the matrices \mathbf{I} and $\mathbf{0}$ are of appropriate dimensions. Next, the quadratic term is lifted by using (29)

$$\begin{aligned}
P_2(T) &= \int_0^T -\mathbf{z}(t)^T \mathbf{Q} \mathbf{z}(t) dt \\
&= - \sum_{k=0}^{N-1} \int_0^h \left(\mathbf{z}_k^T e^{\mathbf{A}^T t} + \mathbf{u}_k^T \int_0^t \mathbf{B}^T e^{\mathbf{A}^T(t-s)} ds \right) \mathbf{Q} \\
&\quad \left(e^{\mathbf{A}t} \mathbf{z}_k + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B} ds \mathbf{u}_k \right) dt \\
&= - \sum_{k=0}^{N-1} \int_0^h \begin{bmatrix} \mathbf{z}_k^T & \mathbf{u}_k^T \end{bmatrix} e^{\tilde{\mathbf{A}}^T t} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{Q} \\
&\quad \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}t} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} dt \\
&= - \sum_{k=0}^{N-1} \begin{bmatrix} \mathbf{z}_k^T & \mathbf{u}_k^T \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \mathbf{Y}(h) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} \tag{32}
\end{aligned}$$

where $\tilde{\mathbf{A}}$ is as above and

$$\mathbf{Y}(h) = \int_0^h e^{-\tilde{\mathbf{A}}^T(h-t)} \bar{\mathbf{Q}} e^{-\tilde{\mathbf{A}}(h-t)} dt \quad (33)$$

$$\bar{\mathbf{Q}} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{Q} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$$

The integral in (33) is the solution to a matrix differential equation

$$\mathbf{Y}(h) = \int_0^h e^{-\tilde{\mathbf{A}}^T(h-t)} \bar{\mathbf{Q}} e^{-\tilde{\mathbf{A}}(h-t)} dt \Rightarrow$$

$$-\frac{d}{dh} \mathbf{Y}(h) = \tilde{\mathbf{A}}^T \mathbf{Y}(h) + \mathbf{Y}(h) \tilde{\mathbf{A}} - \bar{\mathbf{Q}}, \quad \mathbf{Y}(0) = \mathbf{0}. \quad (34)$$

Using the $\text{Vec}(\cdot)$ notation which is defined as

$$\text{Vec}(\mathbf{P}) = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, \quad (35)$$

where p_i is the columns of \mathbf{P} , it is possible to formulated (34) as

$$-\frac{d\text{Vec}(\mathbf{Y}(h))}{dh} = \mathbf{F} \text{Vec}(\mathbf{Y}(t)) - \text{Vec}(\bar{\mathbf{Q}}) \quad (36)$$

where

$$\mathbf{F} = \left(\mathbf{I} \otimes \tilde{\mathbf{A}}^T + \tilde{\mathbf{A}}^T \otimes \mathbf{I} \right)$$

and \otimes denotes the Kronecker product. By using the solution to standard vector differential equation and (30), the solution to (36) is given by

$$\begin{aligned} \text{Vec}(\mathbf{Y}(h)) &= \int_0^h e^{\mathbf{F}(h-\tau)} d\tau \text{Vec}(\bar{\mathbf{Q}}) \\ &= e^{\mathbf{F}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{F}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \text{Vec}(\bar{\mathbf{Q}}) \\ &= e^{\mathbf{F}h} \tilde{\mathbf{F}} \text{Vec}(\bar{\mathbf{Q}}), \end{aligned}$$

where

$$\tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{F}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \hat{\mathbf{F}} = \begin{bmatrix} \mathbf{F} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

That is (32) can be expressed as

$$P_2(T) = - \sum_{k=0}^{N-1} \begin{bmatrix} \mathbf{z}_k^T & \mathbf{u}_k^T \end{bmatrix} \begin{bmatrix} \mathbf{N}_{zz} & \mathbf{N}_{zu} \\ \mathbf{N}_{uz} & \mathbf{N}_{uu} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} \quad (37)$$

where

$$\begin{aligned}
 N_{zz} &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \text{Vec}^{-1} \left(e^{\mathbf{F}h} \tilde{\mathbf{F}} \text{Vec}(\bar{\mathbf{Q}}) \right) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \\
 N_{zu} &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \text{Vec}^{-1} \left(e^{\mathbf{F}h} \tilde{\mathbf{F}} \text{Vec}(\bar{\mathbf{Q}}) \right) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \\
 N_{uz} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \text{Vec}^{-1} \left(e^{\mathbf{F}h} \tilde{\mathbf{F}} \text{Vec}(\bar{\mathbf{Q}}) \right) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \\
 N_{uu} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \text{Vec}^{-1} \left(e^{\mathbf{F}h} \tilde{\mathbf{F}} \text{Vec}(\bar{\mathbf{Q}}) \right) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}
 \end{aligned}$$

with $\text{Vec}^{-1}(\cdot)$ denoting the inverse of the Vec-operator in (35), i.e., reshaping the vector into a matrix.

Technical Note

Alternative Problem Formulations

Martin Kragelund

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The layout has been revised

In this note five difference formulations/implementations of the problem below are considered. The original problem formulation is as follows

$$\begin{aligned} \max_{u \in U} \quad & P(T) = \int_0^T g_P(z, t) dt \\ \text{subject to} \quad & \dot{z} = \mathbf{A}z + \mathbf{B}u, \end{aligned} \quad (1.1)$$

and moreover, $y_e(z)$ should track a predefined reference signal, $y_r(t)$. Depending on how the reference tracking is formulated, different (discrete-time) optimization problems arise.

The profit growth function, g_P , can when $y_e \approx y_r$ be approximated by

$$g_p(z, t) = \Theta(t)z + \varphi(t),$$

where

$$\begin{aligned} \Theta(t) &= p_{R1}(t)\gamma^T \mathbf{Q} - p_{R2}(t)p_C^T \mathbf{C} + \vartheta(t), \\ \varphi(t) &= p_{R1}(t)\gamma^T \mathbf{b} + p_{R2}(t)\zeta(t), \end{aligned}$$

and $\vartheta(t)$ and $\zeta(t)$ makes up for the switching function in original formulation of the controllability, i.e.,

$$\begin{aligned} \vartheta(t) &= \begin{cases} 0 & y_r(t) \in S_1 \\ \frac{\xi^T \mathbf{Q}}{y_r(t)} & y_r(t) \in S_2 \\ 0 & y_r(t) \in S_3, \end{cases} \\ \zeta(t) &= \begin{cases} 0.133 & y_r(t) \in S_1 \\ \frac{\xi^T \mathbf{b}}{y_r(t)} & y_r(t) \in S_2 \\ 0.133 & y_r(t) \in S_3, \end{cases} \end{aligned}$$

The time period T is divided into N equally sized time units, h , i.e., $T = Nh$. It is assumed that $\Theta(t)$, $\varphi(t)$, $\psi(t)$, $y_r(t)$ can be approximated by piecewise constant functions for each time step, i.e.,

$$\begin{aligned} \Theta(t) &= \Theta_k, & kh < t < (k+1)h, \\ \varphi(t) &= \varphi_k, & kh < t < (k+1)h, \\ y_r(t) &= y_{rk}, & kh < t < (k+1)h. \end{aligned}$$

Furthermore, the control will be assumed piecewise constant as customary when digital to analogue conversion is performed using sample-and-hold circuits.

Using a fact from [1] the continuous time state $z(t)$ in the dynamic system in (1.1) can be described by

$$\begin{aligned} z(t) &= e^{\mathbf{A}t} z_0 + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B}u_0(s) ds \\ &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \exp \left\{ \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} t \right\} \begin{bmatrix} z_0 \\ u_0 \end{bmatrix}, \end{aligned} \quad (1.2)$$

where \mathbf{I} is an identity matrix with appropriate dimension. Using (1.2) it is possible to derive the following formula which is used during the discretization of the cost and constraint.

$$\begin{aligned} \int_0^h e^{\mathbf{A}t} dt &= e^{\mathbf{A}h} \int_0^h e^{-\mathbf{A}(h-t)} dt \\ &= e^{\mathbf{A}h} \left(e^{-\mathbf{A}h} \cdot 0 + \int_0^h e^{-\mathbf{A}(h-t)} \mathbf{I} dt \right) \\ &= e^{\mathbf{A}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \exp \left\{ \begin{bmatrix} -\mathbf{A} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} h \right\} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \end{aligned} \quad (1.3)$$

The objective function, $P(T)$, in the optimization problem in (1.1) is converted to discrete time by using the above, i.e.,

$$\begin{aligned} P(T) &= \sum_{k=0}^{N-1} \int_{kh}^{(k+1)h} (\Theta(t) \mathbf{z}(t) + \varphi(t)) dt \\ &= \sum_{k=0}^{N-1} \Theta_k \int_0^h \left(e^{\mathbf{A}t} \mathbf{z}_k + \int_0^t e^{\mathbf{A}(t-s)} \mathbf{B} ds \mathbf{u}_k \right) dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \Theta_k \int_0^h \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}t} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} + h\varphi_k, \end{aligned}$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} -\tilde{\mathbf{A}} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

The optimization problem can now be formulated as

$$\begin{aligned} \max_{\mathbf{u}_k \in U} \quad & \sum_{k=0}^{N-1} \mathbf{C}_k \mathbf{z}_k + \mathbf{D}_k \mathbf{u}_k + \mathbf{E}_k \\ \text{subject to} \quad & \mathbf{z}_{k+1} = \Phi \mathbf{z}_k + \Gamma \mathbf{u}_k, \end{aligned}$$

where

$$\begin{aligned} \mathbf{C}_k &= \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \\ \mathbf{D}_k &= \Theta_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \\ \mathbf{E}_k &= h\varphi_k, \Phi = e^{\mathbf{A}(t_{k+1}-t_k)}, \text{ and } \Gamma = \int_0^{t_{k+1}-t_k} e^{\mathbf{A}s} ds \mathbf{B} \end{aligned}$$

when the tracking is disregarded.

When considering the reference tracking different approaches can be used to formulate them. In this note three different methods are considered, which will be described briefly below.

Hard Constraint: In this approach the tracking is formulated as a constraint in the optimization such that the reference is followed within a reference band.

Norm u: In this approach the tracking is formulated as above, where the constraint is only guaranteed at certain times and then the deviation of the new control from the old control is minimized. The advantages of this approach is that the discrete time formulation has much less constraints which might speed up the optimization and the limit on the new control limits “bang-bang” behavior as will be seen later.

Quadratic: In this approach the tracking constraint is included in the profit function as a norm of the difference between the efficiency and the reference and thus penalizing deviations. This approach also has few constraints in the discrete times implementation but does, however, also introduce quadratic terms to the objective function.

Furthermore, the method including a reference band will be considered in three different discrete formulations, where the difference is how rigorous the tracking constraint is guaranteed fulfilled. This serves as intermediate steps between “hard constraint” and “norm u”.

1 Hard Constraint

One of the approaches is formulated by tracking the reference within a reference band α which in continuous time is defined as

$$h(\mathbf{z}(t), t) \geq 0, \quad (1.4)$$

where

$$h(\mathbf{z}(t), t) = \Upsilon \mathbf{z}(t) + \psi(t), \quad (1.5)$$

with

$$\begin{aligned} \Upsilon &= \begin{bmatrix} \gamma^T \mathbf{Q} \\ -\gamma^T \mathbf{Q} \end{bmatrix}, \\ \psi(t) &= \begin{bmatrix} \gamma^T \mathbf{b} - y_r(t) + \alpha \\ -\gamma^T \mathbf{b} + y_r(t) + \alpha \end{bmatrix}. \end{aligned}$$

The discrete time approximation yields

$$\Psi_l \mathbf{z}_k + \Pi_l \mathbf{u}_k + \Omega_{k,l} \geq 0$$

where for $l = 0, 1, 2, \dots, L$

$$\begin{aligned} \Psi_l &= \Upsilon e^{\mathbf{A} \frac{l-1}{L} h}, \\ \Pi_l &= \Upsilon \int_0^{\frac{l-1}{L} h} e^{\mathbf{A}(\frac{l-1}{L} h - s)} \mathbf{B} ds, \\ \Omega_{k,l} &= \psi(\frac{l-1}{L} h + kh). \end{aligned}$$

The parameter, L , is chosen such that the constraint is guaranteed to be satisfied between sampling of the system. In this work $L = 5$ is chosen for the “hard constraint.”

In Figure 1.1

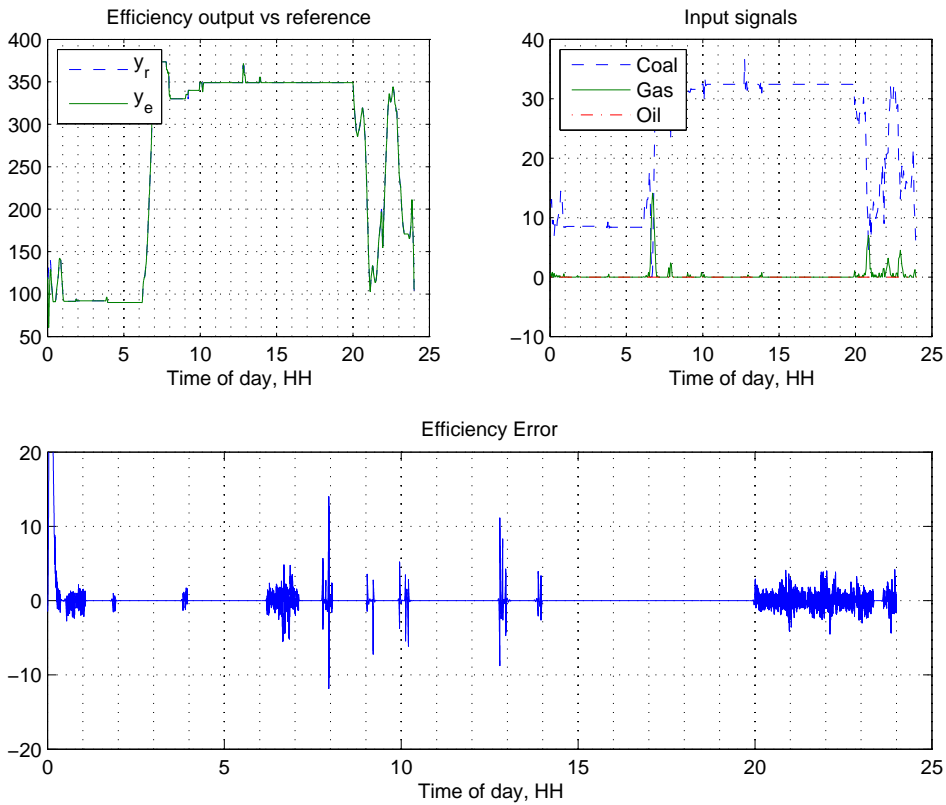


Figure 1.1: Hard Constraint - intersampling samples = 5.

2 Loose Constraint 1

This approach is similar to the above where the sampling of the tracking constraint, $h(z(t), t)$, is changed. The constraint is only sampled once for each sampling of the system ($L = 1$) and then the compliance with the constraint is verified a posteriori through simulation. Using the derivation in the previous section the discrete tracking constraint in this approach yields

$$\Upsilon z_k + \psi_k \geq 0. \quad (1.6)$$

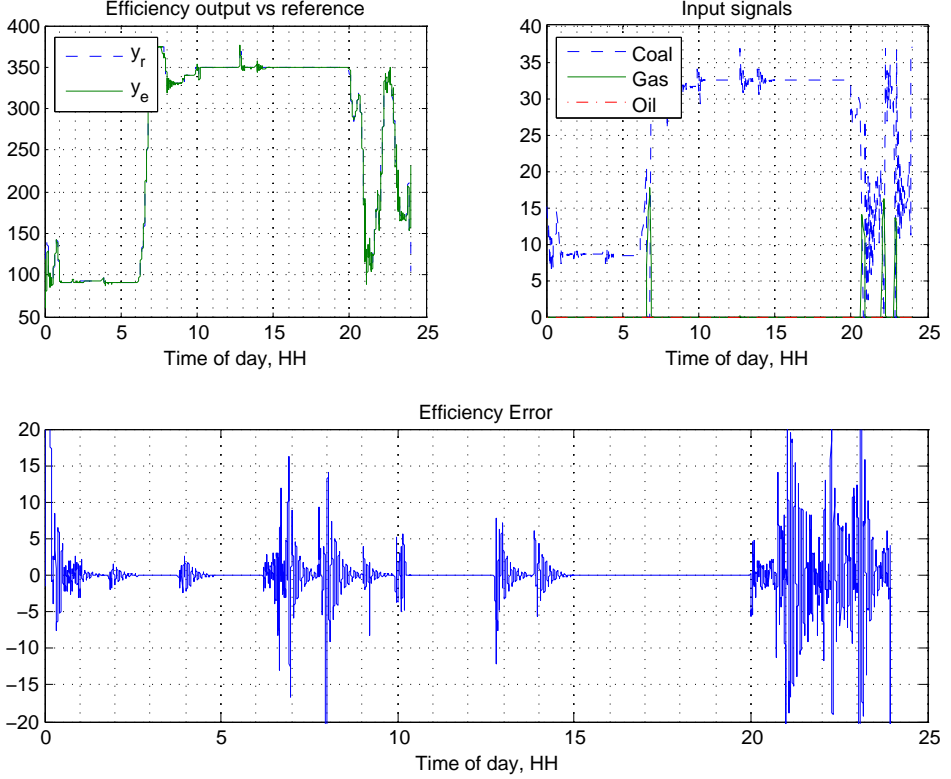


Figure 1.2: Loose Constraint - intersampling samples = 1.

3 Loose Constraint 2

As above, where even less samples are used in the tracking constraint. Furthermore, the α band is removed and exact tracking is imposed at the evaluation points. The discrete tracking constraint is formulated as

$$\gamma^T Q z_l - y_{r_l} = 0, \quad l \in \{0, 5, 10, \dots, N\}. \quad (1.7)$$

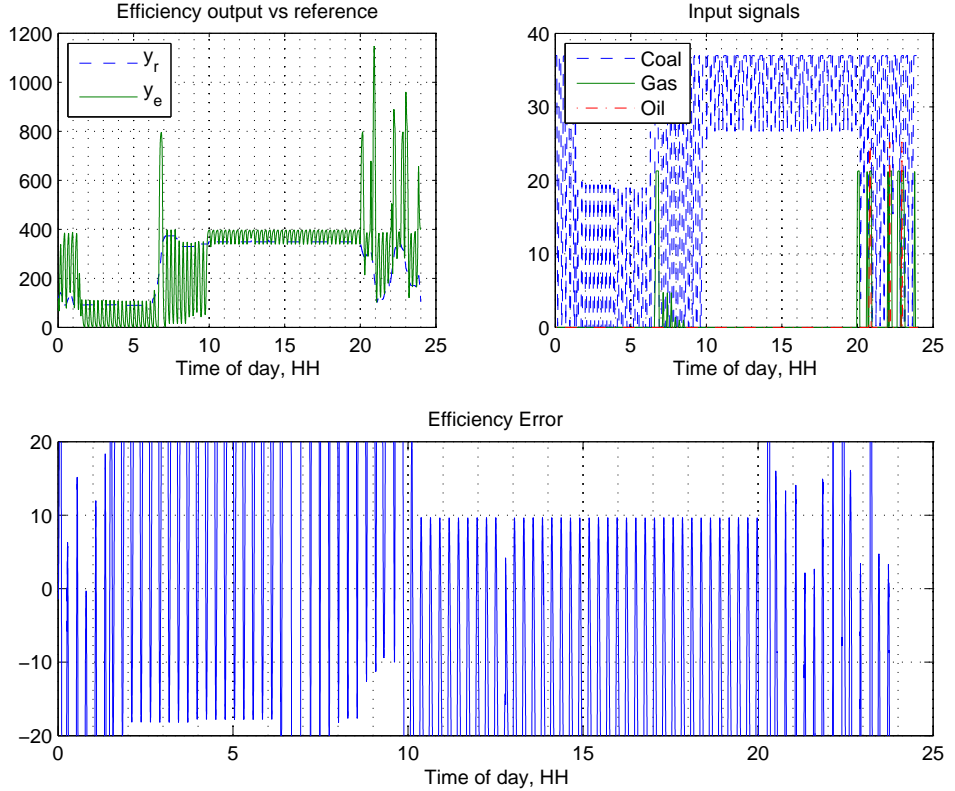


Figure 1.3: Loose Constraint - intersampling samples = 0.2.

4 Norm u

In this approach the reference tracking is formulated as above, however, deviations in the controls are penalized in the objective functions by including a norm, i.e., the term

$$N(T) = -\beta_n \int_0^T \left| \frac{d\mathbf{u}}{dt} \right| dt$$

is added to $P(T)$ in (1.1)

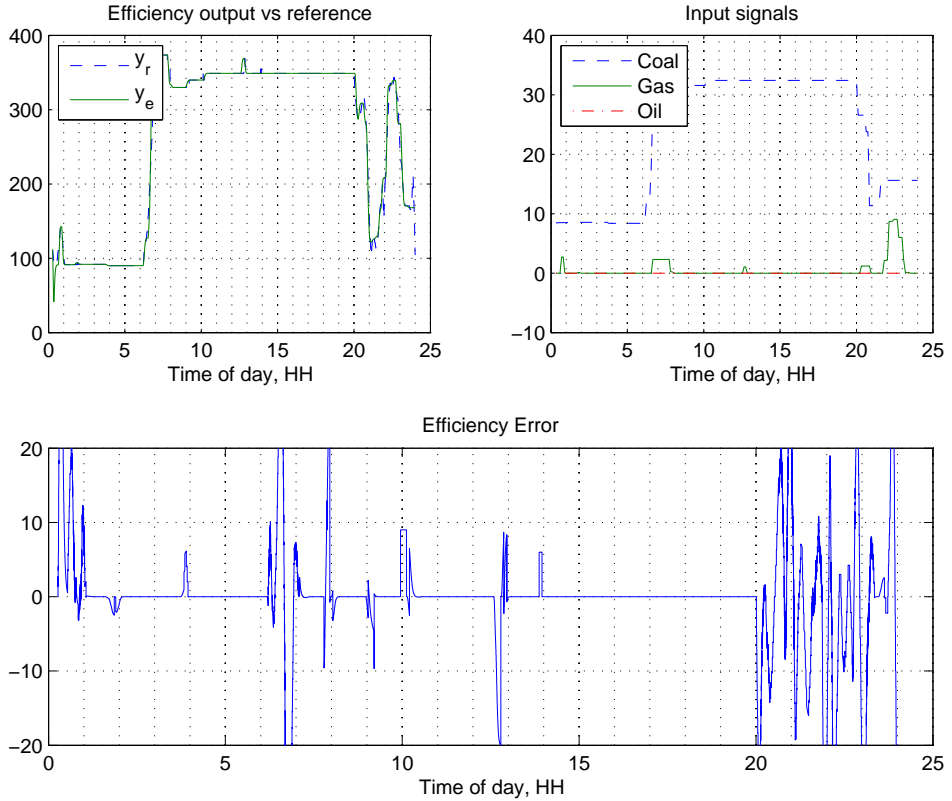


Figure 1.4: Norm on $u_k - u_{k-1}$.

5 Quadratic

In this section we will include the reference tracking as a cost on the deviation from the reference, i.e., the cost will be expanded by the term

$$Q(T) = -\beta_q \int_0^T \|\gamma^T Q z(t) - y_r(t)\|^2 dt, \quad (1.8)$$

where $\|\cdot\|$ is the Euclidean norm.

In the following we will apply lifting to obtain a discrete version of $Q(T)$.

$$\begin{aligned} Q(T) &= -\beta_q \int_0^T \|\gamma^T Q z(t) - y_r(t)\|^2 dt \\ &= -\beta_q \int_0^T \left(z(t)^T Q^T \gamma \gamma^T Q z(t) - 2y_r(t) \gamma^T Q z(t) + y_r(t)^2 \right) dt \\ &= -\beta_q \sum_{k=0}^{N-1} \int_0^h \left(z_k^T e^{A^T t} + u_k^T \int_0^t B^T e^{A^T(t-s)} ds \right) Q^T \gamma \gamma^T Q (e^{A t} z_k \\ &\quad + \int_0^t e^{A(t-s)} B ds u_k) dt - \sum_{k=0}^{N-1} 2y_{rk} \gamma^T Q \int_0^h \left(e^{A t} z_k + \int_0^t e^{A(t-s)} B ds u_k \right) dt \\ &\quad + h \sum_{k=0}^{N-1} y_{rk}^2 \\ &= -\beta_q \sum_{k=0}^{N-1} \int_0^h \begin{bmatrix} z_k^T & u_k^T \end{bmatrix} e^{\tilde{A}^T t} \begin{bmatrix} I \\ 0 \end{bmatrix} Q^T \gamma \gamma^T Q \begin{bmatrix} I & 0 \end{bmatrix} e^{\tilde{A} t} \begin{bmatrix} z_k \\ u_k \end{bmatrix} dt \\ &\quad - \sum_{k=0}^{N-1} 2y_{rk} \gamma^T Q \int_0^h \begin{bmatrix} I & 0 \end{bmatrix} e^{\tilde{A} t} \begin{bmatrix} z_k \\ u_k \end{bmatrix} dt + h \sum_{k=0}^{N-1} y_{rk}^2 \\ &= -\beta_q \sum_{k=0}^{N-1} \begin{bmatrix} z_k^T & u_k^T \end{bmatrix} e^{\tilde{A}^T h} \int_0^h e^{-\tilde{A}^T(h-t)} \tilde{C}^T \tilde{C} e^{-\tilde{A}(h-t)} dt e^{\tilde{A} h} \begin{bmatrix} z_k \\ u_k \end{bmatrix} \\ &\quad - \sum_{k=0}^{N-1} 2y_{rk} \tilde{C} e^{\tilde{A} h} \begin{bmatrix} I & 0 \end{bmatrix} e^{\tilde{A} h} \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} z_k \\ u_k \end{bmatrix} dt + h \sum_{k=0}^{N-1} y_{rk}^2 \end{aligned} \quad (1.9)$$

where

$$\hat{A} = \begin{bmatrix} -\tilde{A} & I \\ 0 & 0 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \quad \tilde{C} = \gamma^T Q \begin{bmatrix} I & 0 \end{bmatrix} \quad (1.10)$$

The integral left in (1.9) is on the form of the solution to a matrix differential equations, i.e.,

$$\begin{aligned} Y(t) &= \int_0^t e^{F^T(t-\tau)} H e^{F(t-\tau)} d\tau \Rightarrow \\ \dot{Y}(t) &= F^T Y(t) + Y(t) F + H, \quad Y(0) = 0. \end{aligned} \quad (1.11)$$

Using the $\text{Vec}(\cdot)$ notation which is defined as

$$\text{Vec}(\mathbf{P}) = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, \quad (1.12)$$

where p_i is the columns of \mathbf{P} , it is possible to formulated (1.11) as

$$\frac{d\text{Vec}(\mathbf{Y}(t))}{dt} = \underbrace{(\mathbf{I} \otimes \mathbf{F}^T + \mathbf{F}^T \otimes \mathbf{I})}_{\tilde{\mathbf{F}}} \text{Vec}(\mathbf{Y}(t)) + \text{Vec}(\mathbf{H}) \quad (1.13)$$

which yields the solution

$$\begin{aligned} \text{Vec}(\mathbf{Y}(t)) &= \int_0^t e^{\tilde{\mathbf{F}}(t-\tau)} d\tau \text{Vec}(\mathbf{H}) \\ &= e^{\tilde{\mathbf{F}}t} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{F}}t} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \text{Vec}(\mathbf{H}), \end{aligned} \quad (1.14)$$

where

$$\hat{\mathbf{F}} = \begin{bmatrix} -\tilde{\mathbf{F}} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (1.15)$$

Inserting this in (1.9) yields

$$\begin{aligned} Q(T) &= -\beta_q \sum_{k=0}^{N-1} \begin{bmatrix} \mathbf{z}_k^T & \mathbf{u}_k^T \end{bmatrix} e^{\tilde{\mathbf{A}}^T h} \text{Vec}^{-1} \left(e^{\tilde{\mathbf{F}}t} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{F}}t} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \text{Vec}(\mathbf{H}) \right) e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} \\ &\quad - \sum_{k=0}^{N-1} 2y_{r_k} \tilde{\mathbf{C}} e^{\tilde{\mathbf{A}}h} \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} e^{\hat{\mathbf{A}}h} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}_k \\ \mathbf{u}_k \end{bmatrix} dt + h \sum_{k=0}^{N-1} y_{r_k}^2, \end{aligned}$$

where $\text{Vec}^{-1}(\cdot)$ denotes the inverse of the Vec -operator in (1.12), i.e., reshaping the vector into a matrix.

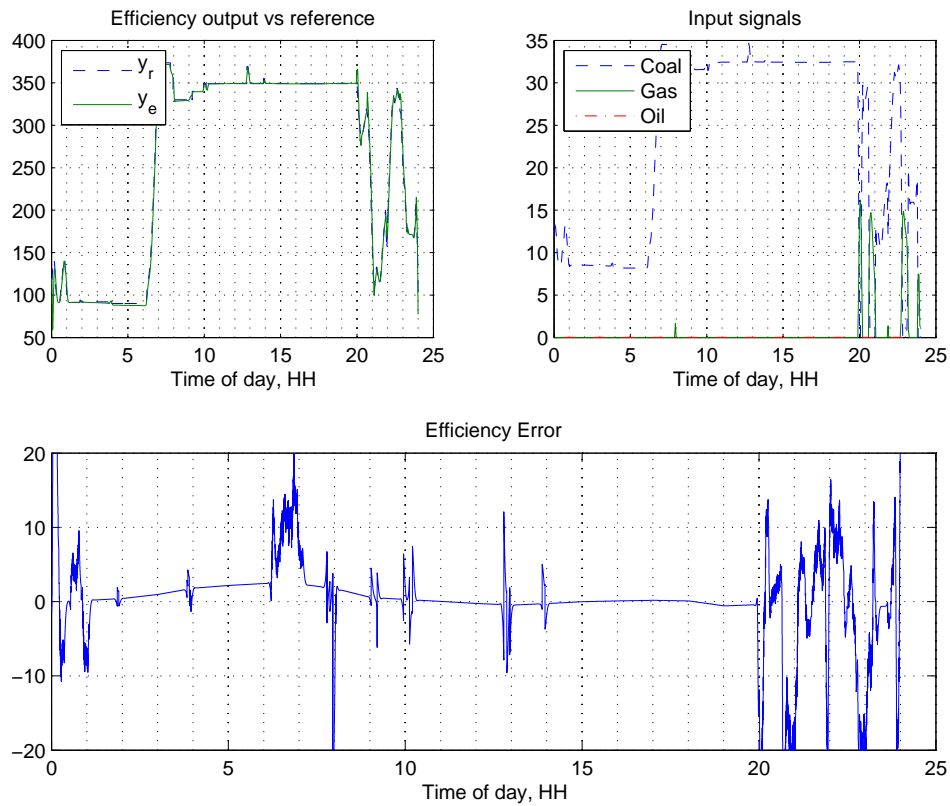


Figure 1.5: Quadratic.

6 Comparison of Optimization Time

In table 1.1 the times for preprocessing, formulating the objective function, running the optimization, and used in by the solve are presented. The method “ICCA” refers to the

Method	Preprocess.	Objective	Opt.	Solve time ¹	Total	Solver
ICCA	30.83 s	0.03 s	865.1 s	-	896 s	SeDuMi
loose 1	31.6 s	0.02 s	136.4 s	-	168 s	SeDuMi
loose 2	28.35 s	0.03 s	6.3 s	6.03 s	34.7 s	SeDuMi
norm u	28.13 s	0.02 s	5.1 s	4.7 s	33.3 s	SeDuMi
quadratic	27.3 s	102.6 s	2.2 s	0.33 s	132.1 s	BPMPD
quadratic	27.3 s	50.4 s	79.3 s	47.3 s	157 s	BPMPD

Table 1.1: Comparison of optimization times between the five alternative formulations.

method used in [2] and has also been explained in Section 1. The methods “loose 1” and “loose 2” are two relaxations of the ICCA method and are described in Section 2 and 3 respectively. To remedy some of the fluctuations in the reference (and inputs) as observed by the “loose 2” method a constraint is put on the input signal changes. This is denoted the “norm u” method and presented in Section 4. Finally, in the “quadratic” method (see Section 5 the tracking is included in the objective function as a penalty on the quadratic tracking error. The reason for this method is presented twice in the table above is due to different implementations of the interface between the solve BPMPD and YALMIP. By examining the total optimization time in the table above and relating the tracking of the different methods it is concluded that the quadratic method should be investigated further.

References

- [1] K. J. Åström and B. Wittenmark, *Computer-Controlled Systems - Theory and Design*. Prentice-Hall International, 1990.
- [2] M. Kragelund, U. Jönsson, J. Leth, and R. Wisniewski, “Optimal production planning of a power plant,” in *Proceedings of the International Conference on Control and Automation, Christchurch*, 2009.

¹Solve time is given by YALMIP and is the time spend by the call of the solver used.